STUDIES RELATING TO THE INFLUENCE OF TOPOGRAPHICAL FEATURES UPON SURFACE AIR-FLOW AND INCIDENT RADIATION

A THESIS SUBMITTED BY

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GLOSSARY OF TERMS AND SYMBOLS.

Some terms, and the more important of those symbols which occur frequently, are defined below.

With respect to symbols, customary usage has been adopted as far as possible; in consequence a particular symbol can assume different meanings in the different Parts.

The entries (I) (II) or (III) indicate the Part(s) in which a given term or symbol has the stated meaning assigned to it.

Barrier	(I)	(Wind)-barrier: any obstruction to air flow.
"Characteristic" Circle	(II)	A circle, centred halfway between the origin O and the end A of the vector representing the vector mean wind of a normal circular wind distribution, such that the intercept of any radius from O with the circumference gives the mean speed in the direction of the radius.
"Corresponding" Points	(III)	Points on the surface of the globe at which a horizontal plane is parallel to a sloping plane at some selected point.
Displacement	(II)	Alternatively "flow" or "run-of-wind": total movement of air in a given period past a fixed point as registered by an anemometer head free to rotate about a vertical axis.
"Efficiency"	(I)	(of a wind barrier). The degree or intensity of the influence exerted on wind speed (or generally any meteorological variable) by a wind barrier: expressed either in absolute

		terms, or, more generally, relative to conditions outside the "zone of influence" (see over): either integrated over the whole "zone", or as experienced at a particular point within the "zone", or with respect to the maximum degree of influence exerted.
Permeability	(I)	(of a barrier). The ratio (almost invariably expressed as a percentage) of the total unblocked to the total area of a wind-barrier, the areas being projected on a vertical plane, parallel to the main axis of the belt.
Relative wind-speed	(I)	The ratio (almost invariably expressed as a percentage) of the wind-speed within the "zone of influence" of a barrier, to the speed of the undisturbed or "free" wind. Measuring points in the disturbed and undisturbed streams should strictly be at the same height above the surface.
Shelter-belt	(I)	Barrier formed by a stand of trees - typically of length many multiples of its height (H) and frequently having width (or thickness) some multiples of H.
Shelter-screen	(I)	A barrier - typically of simple geometrical form and uniform structure: generally only a few feet high, and constructed of wood or similar materials. Barriers used in wind-tunnel studies.

"Zone of Influence"	(I)	(of a wind-barrier). The distance downwind from a barrier; or the area in vertical or horizontal section; in which the wind-speed (or, in general, any climatological variable) is affected to some predetermined extent, by the barrier. All linear dimensions are expressed in terms of H.

In Part II parameters of the actual wind-distributions are enclosed in curved brackets ( ); and those of a normal circular distribution (designated NCD) defined by reference to some chosen parameter of the actual distribution (generally q) enclosed in square brackets [ ].

A	(II)	The end A of the vector $\overrightarrow{OA}$ representing the vector mean wind of a two-dimensional wind distribution.
D; D(V); D(ϕ); D(V,ϕ)	(II)	Total "flow" or displacement past a point within some stated period; displacement for "all speeds" 0 to V (and hence independent of direction); that for "all directions"; and that for winds occurring within a segment of the wind field of which the mid-point is defined by (V, ϕ).
F; F(V); F(ϕ); F(V,ϕ)	(II)	Frequency of wind observations: the suffixes having the same meanings as for D, D(V) etc.
h	(I)	General height of a tree canopy etc.
H	(I)	Height of a wind barrier.

q	(II)	The ratio of the module of the vector mean wind and the scalar mean wind. Almost invariably expressed as a percentage. The "constancy" of Brooks et al. (1950).
R	(II)	The radius of the "characteristic" circle.
s	(II)	Standard deviation of wind speeds.
S, S'..	(III)	Typical points on the surface of the globe.
T, T'..	(III)	The "corresponding" points for S, S'..
t	(III)	Hour angle of the sun.
u; $\overline{u}(z)$; $u_{(0)}$	(I)	Wind speed, mean wind speed at height above the surface; mean speed of the undisturbed flow along a horizontal axis usually normal to the line of a barrier.
v	(II)	The magnitude of the vector deviation of a wind from a vector mean wind.
$V_{(g)}$. $V_{(s)}$	(I)	Geostrophic and surface wind-speeds.
$V_{(s)}$ $\overline{V}_{(s)}$	(II)	Wind speed, mean wind-speed.
$V(\phi)$; $V(\psi)$	(II)	Wind speed in the direction indicated by the suffix.
$\mathbf{V}$ $\overline{\mathbf{V}}$ $\lvert\overline{\mathbf{V}}\rvert$	(II)	Wind vector, vector mean wind, magnitude of vector mean wind.
z_o	(I)	"Roughness length", a parameter in the wind profile laws, related to the linear dimensions of surface roughness elements.
α	(III)	Angle between the line of greatest slope of a plane surface and the meridian, viz. the "aspect" angle of a sloping plane.

β	(III)	Angle, with respect to a horizontal plane, of the line of greatest slope of an inclined plane.
δ	(III)	Solar declination.
ε	(I)	A linear quantity specifying, and derived directly from, the heights of the roughness elements on a surface.
ε	(III)	Difference of longitude between point S and its corresponding point T.
$\theta(S)$, $\theta(T)$	(III)	Angle between the sun's beam and the normals to plane surfaces at S and T.
$\sigma_{(0)}$	(II)	Standard vector deviation (the suffix is dropped in expressions such as $\sigma/\lvert\bar{V}\rvert$, i.e. when there is no risk of ambiguity.
$\sigma_{(x)}$ $\sigma_{(y)}$ $\sigma_{(N)}$ etc.	(II)	Standard deviation of wind components along axes indicated by the suffixes.
ϕ	(I)	Geometrical permeability of a wind-barrier (i.e. 1 - (blockage ratio)), usually expressed as a percentage.
ϕ	(II)	Angle of wind vector from north (clockwise rotation positive in accordance with climatological convention).
ϕ	(III)	Latitude of point S.
ψ	(II)	Angle between wind vector and the direction of the vector mean wind.
ψ	(III)	Co-latitude of point S.

Chapter 1
Introduction

Section I. The general background to the problem

The present studies aim to contribute towards a solution of one of the central problems of agricultural meteorology, viz. the specification and classification of climate and weather on a scale intermediate between that adopted in regional analyses - where areas measured in many hundreds of square miles are under consideration -, and the now vigorously explored realm of micrometeorology with its main emphasis upon the energy exchanges in, and the physical properties of, a layer of atmosphere some tens of metres deep lying over a plane surface with or without a uniform cover of low vegetation. Terms such as "local -", "meso -" and, latterly, "topo -" climatology (Thornthwaite, 1954) embrace the subject matter now under consideration, and in spite of the etymological claims of the prefix "meso", "topo" is probably the most illuminating as it forces attention upon the role of topographical features, especially the geometrical form of the earth's surface, in influencing air-flow and the exchange of radiant energy.

The writer's particular interest has been in connection with the design and siting of shelter-belts and shelter-screens for reducing certain environmental stresses upon agricultural crops and farm livestock. This is not the place to consider the evidence for the existence of such stresses nor the desirability of reducing them in order to increase production - for a discussion of these aspects reference might be made to other papers by the writer (Gloyne, 1955, 1957). What is relevant is that reliable advice is now dependent upon more knowledge of the detail of "local" climate and weather in the absence of any imposed shelter, particularly in hilly areas; for, as emphasised in a recent authoritative treatise on the subject, "Shelter Belts and Micro-climate" (Caborn, 1957), "a comprehensive scheme of research is desirable (in which) meteorology can contribute valuable information on the structure of the climate near the ground and especially on the pattern of air-flow in regions of broken relief".

There is, of course, no lack of studies of local climate, ranging from extensive monographs, e.g. Seltzer (1934) through specialised investigations such as that of the topographical obstruction to direct sunshine in Alpine valleys (Garnett, 1937), to accounts of the weather peculiarities of individual airfields (Durst, 1949).

The examination of land and sea breezes has received much attention (see Sutton, 1953, p.267, Pearce, 1955), and approximate mathematical solutions have been obtained "for the well-defined local circulations (which) are found in valleys leading into mountain ranges from plains" (Sutton, loc.cit.). A particular example of considerable interest is dealt with in a paper by Hewson (1945), in which it is shown how knowledge of local wind circulations can aid the effective control of pollution generated by industrial plant in well marked valleys. In problems of this type the orientation of the direct solar beam with respect to mountain slopes is of prime importance.

To realise fully the potential value of such investigations, it is obviously necessary to seek ways of placing the knowledge obtained in a general framework, and it is here suggested that any synthesis will involve a study of weather and climate in relation to surface geometry in the widest sense. This, if true, implies that methods are needed for defining the geometry of an area in such terms and on such scales as are meaningful in relation to certain climatic parameters, which in turn have been formulated in the light of the sensitivity of some operation or body to the atmospheric environment. In view of the obvious importance of the quantitative description of ground contour, reference will be made to certain techniques utilised by geographers (see Appendix I(a)).

To prevent any misunderstanding, it must be emphasised that there is no suggestion that a general theoretical solution can be reached concerning the interaction of the atmosphere with the small and medium-sized topographical features of the earth's surface. All that is envisaged is that, given tolerance ranges for wind, temperature, humidity etc., it might be possible progressively to limit the uncertainty in the estimation or "forecasting" of these variables by increased attention to surface geometry. Further, should more precise information be required in any given case, a prior examination of the problem from the standpoint outlined above might well minimise the amount of on-site field work required, and maximise the value of information obtained from field surveys, both singly and collectively. At the very least, attention to the topic might be expected to point the way to ad-hoc methods of codifying the increasing accumulation of climatological data.

where?

Section 2. Aspects of particular relevance to agriculture and allied industries.

In this section it is proposed to draw extensively upon two memoranda prepared by the writer; one for the Meteorological Research Committee already mentioned (Gloyne, 1957) and the second (Gloyne, 1958) for the "Joint Committee" of the Agricultural Improvement and Agricultural Research Councils.

In both documents the impact of certain meteorological variables (particularly air-flow) upon specific phases of agriculture, horticulture and forestry was under consideration. Stress was laid on the need for quantitative estimates of significant limits, thresholds, etc., note was taken of work in progress, and proposals were put forward for future work. The end-point of the whole project turns upon the need for, and techniques of providing, a degree of protection from wind, rain, cold and solar radiation by the provision of shelter-belts ranging from forestry plantations to the portable screens of netting and similar materials used in intensive horticulture. Any shelter element gives rise to some redistribution of radiant energy which, together with the imposed alteration in air-flow, brings consequential changes in most other properties of the physical environment. Obviously the meteorological aspect is but one facet of the problem, although it is the one common element evident in all its various forms: equally important are parallel studies of animal and plant ecology and of the numerical and cartographic representation of the earth's surface.

It was found helpful to consider separately the problem as it arises:

(i) in horticulture, and in the design, siting and management of structures ranging from low glass to farm buildings;

(ii) in agriculture, both as regards crops and livestock;

(iii) in forestry;

associated respectively with increasing permissible tolerances in the relevant meteorological information;

and (iv) a group of topics straddling the whole range, and relating to the transport, diffusion and interception of airborne material.

In all cases the solution involves certain definite steps: firstly, the examination of evidence that a weather hazard does in fact exist; secondly, the assessment of the liability of the area in question to these weather hazards; thirdly, the appraisal of methods aimed at either reducing the hazard as with shelter-belts, buildings, etc., or at circumventing it by adopting different crops, stock, or farming practices; and finally, the consideration of the economics of adopting any course of action - a stage in which the analysis of contingencies may well involve meteorological factors.

(i) Horticultural Aspects. Information is required on the distribution of wind in speed and direction from the surface to some tens of feet above the ground at any given place, further classified according to season, temperature, precipitation etc. Given a particular system of shelter elements of stated dimensions and permeabilities, we require to know what effect these would have on the incident wind, what would be the consequential effects due to the shading by the barrier(s), and how the ventilation and heat loss of downwind structures would be altered.

Analyses of the extensive literature on interrupted flow on the "horticultural" scale have been made by the writer (1953 and 1955) and some advice of practical value may now be given - at any rate to commercial holdings on level or gently undulating countryside - assuming that the basic climatological data exists. At present, however, the anemometric network is deficient even in some intensive horticultural areas, viz. mid- and upper Clyde valleys, Strathmore and East Lothian in Scotland, in many areas of western, midland and northern England and in much of Wales. Further deficiencies are the relative lack of suitably analysed and consolidated data on wind, and the sparse amount of information on wind in combination with other elements.

The influence of upwind barriers on the ventilation of structures is amenable to laboratory investigation, e.g. Irminger and Noddentved (1936), whilst even short qualitative ad-hoc studies in a wind tunnel have proved helpful when considering the design and protection of glasshouses (Hogg, 1958(a)).

(ii) Agricultural Aspects. Arable farming poses questions closely similar to those arising in horticulture, whilst those encountered in livestock farming and forestry have much in common. Little need be added therefore under this head, save to point out that the freedom of posture and of movement of livestock and their varying liability with age, physiological condition and so on to weather "stress", implies that a definite meteorological contribution is possible only after the biological criteria have been determined.

(iii) Forestry. Reliable information on the climate of typically hilly and often remote regions provisionally selected for afforestation would materially assist in the rational planning of the enterprise. A more limited question relates to liability to gale damage, for which a statistical statement of contingencies in any given area is necessary. The integration of forestry and agriculture will be assisted by adequate data on the interactions between the forest and the climate of the adjacent area; on this particular topic Caborn's treatise (1957) provides the fullest information to data relating to conditions in the British Isles.

(iv) Interception, Transport and Deposition of Airborne Material. Under this head may be placed questions relating to the location of upwind sources of airborne spores responsible for "black rust" in wheat (Hogg, 1958(b)) - involving the study of air trajectories up to about ten thousand feet - through questions of soil blowing and snow control, to the small scale phenomena exhibited in the interception of windborne material by leaf surfaces, twigs, etc.

Measures to control the wind erosion of soil and the drifting of snow can now be devised with reasonable precision owing to the growing knowledge of the physics of such processes, and in recent years, investigations by Gregory

(1951), amongst others, on the mechanics of the dispersal of spores and their interception by plant surfaces, have been considerably advanced through the use of the concepts of "collection efficiency" as developed in another context, notably by Glauert (1946), Langmuir and Blodgett (1949).

In view of the growing power of theoretical work on the general problem of diffusion, it seems that at the present time inadequate basic data for any particular instance, i.e. inadequate data on "local" climate, is likely to prove the more troublesome feature to successful practical application.

Experience has shown that progress in resolving practical difficulties on all scales is handicapped by lack of knowledge, suitably codified and presented, on the character of airflow in the following situations:

(i) Over hilly dissected countryside - the phenomenon of "canalisation" or "funnelling" arises repeatedly.
(ii) Over horizontal surfaces with "roughness" elements of various types (crops, trees, houses, hedges etc.) variously disposed.
(iii) Over terrain mentioned in (i), with the addition of the types of obstruction mentioned in (ii).

To date, there appears to be no quantitative definition of "canalisation", and it is hoped that the methods dealt with later in this treatise will be of assistance in this respect. We can, however, state that it is desirable to differentiate between "canalisation" of wind, where some mechanical constraint is envisaged, and mountain and valley winds of the type mentioned earlier (p. 2) in which the differential heating and cooling of mountain slopes plays a dominant role.

As already mentioned, the general solution is unobtainable, but it is reasonable to seek empirical solutions for certain typical, idealised, cases in those weather situations where wind alone or in combination with other factors, is stated to be a limiting factor to production.

Considerable simplification follows from the fact that in many cases only wind speeds > 10 m.p.h. at specimen height will be of relevance (appreciably higher threshold speeds can often be adopted), associated with a continuous cover of more or less heavy cloud, thus implying an adiabatic lapse rate and approximate radiative equilibrium. The important exceptions likely to arise are those of thermally induced winds (e.g. katabatic winds) and instances with clear skies and strong sunshine, where the whole emphasis is on the provision of shade.

Section 3. The plan of the present investigation

In the light of the requirements already set out, an analysis is made of current knowledge relating to air-flow over surfaces with various degrees of roughness, attention being focussed upon results which might eventually make it possible to classify, according to their effects on air-flow, a wide variety of obstructions varying from the uniform low plant cover to certain topographical features. In the course of the investigation, it was judged relevant to review certain, mainly recent, wind-tunnel and field studies on wind-breaks and shelter-belts in the expectation that information on the effect of such obstacles might appreciably clarify views on the operation of more complex systems of obstructions. One outcome of this study was a statement relating to the siting of anemometers which occurs as Appendix I(b).

Scattered information on the control of air-flow by topography and other features of the landscape is then reviewed. The probability that the need for a numerical description of ground contour will emerge as investigations on the lines of the present one develop, led to the preparation of Appendix I(a) which incorporates, what is believed to be, a new approach to the problem.

An efficient meteorological service presupposes that techniques are available for extracting the maximum of value from existing data and for estimating atmospheric conditions at any point in time and space in the absence of direct observations. As a contribution, techniques are developed in Part II which promise to be useful for revealing the existence of, and possibly the extent of, topographical influences on air flow. The general attitude adopted towards the problem has been considerably influenced by the fact that any substantial increase in the observational network is possible only through the use of simple, robust and relatively inexpensive instruments, capable of operating for periods of a week or more without attention and not dependent upon a high voltage electrical supply.

The role of obstacles in intercepting and redistributing radiant energy is of fundamental theoretical importance, as well as having a direct practical bearing upon certain topics arising in agriculture and horticulture, and in view of the regular supply of data on radiation and illumination now coming to hand from a network of stations in the British Isles, the time seemed opportune for a consideration of some methods for handling the information. Accordingly, attention has been given to the derivation of a method of computing the direct radiation intercepted by surfaces of any slope and aspect which was believed to be new, and in many ways superior to those commonly in use. After this particular piece of work was completed, it was found that a similar method had been published in Germany in 1943 by Schütte. However, the co-ordinate system used by Schütte (essentially the terrestrial sphere) differs from that adopted by the writer, and the new system reveals more clearly certain geometrical features of the problem.

The work is completed by a number of appendices in which observational data are summarised, some computational procedures detailed, and topics related to the main thesis discussed.

PART I

SOME GENERAL RELATIONSHIPS
BETWEEN
SURFACE FEATURES AND AIR-FLOW.

Chapter 2

Empirical Relationships between Vertical Wind-speed Profiles and the "Roughness" Characteristics of the Underlying Surface

Section 1. The logarithmic profile law and surface drag

In the now well-established laws governing the variation of mean wind speed with height in the first 50 to 100 metres or so, over surfaces with a uniform cover of short turf, less than about 10 cm. high, surface roughness is allowed for by adopting a single "length" z_0 (the "roughness" length). In the meteorologically important case of fully developed turbulent flow over such a surface in adiabatic conditions the mean wind speed, at height z, $\bar{u}(z)$, is given by

$$\bar{u}(z) = \frac{1}{k}\sqrt{\frac{\tau}{\rho}}\ \ln\frac{z}{z_0} \qquad z > z_0 \qquad 2\ (1)$$

where k = 0.4 (von Karman's constant), ρ the fluid density and τ the shearing stress per unit area ($\equiv \tau_0$ that at the surface for the height range in which Eq.2(1) is valid).

The parameter z_0, which arises formally as a constant of integration, must bear some relationship to the size of the roughness elements, and from earlier measurements of "fully-rough" flow in pipes uniformly roughened with sand grains (Nicuradse, 1933; Schlichting, 1936), it was found that

$$z_0 \simeq \varepsilon/30$$

where ε was the mesh size of the coarser of two sieves through which the sand was sifted.

The general specification of roughness must, however, involve not only a measure of height, but also one dealing with the spacing of the roughness elements. It follows that the usual profile laws are strictly valid only if the roughness elements are so closely (or perhaps so uniformly) disposed that a single length can be said to define the aerodynamic properties of the surface. A further restriction is that only if the obstacles are roughly of the same shape is it justifiable to expect a single mean length to be adequate.

When applied to field conditions, z_0 derived from Eq.2(1) bears varying numerical relationships with any length ε plausibly taken as characteristic of the surface, e.g. average height of grass blades; mean difference between tallest and shortest blades of grass; and for grass surfaces and other low uniform crops z_0 is of the order one quarter of the mean height ε.

In fully rough flow, the drag τ is attributable to pressure differences fore and aft of each obstacle and it is expected that it would be of the form

$$\tau = C_D \cdot \tfrac{1}{2} \rho \bar{u}^2 \qquad 2(2)$$

where C_D is a drag coefficient and $\bar{u}$ the mean speed at some convenient reference height. Some authors use the form

$$\tau = K \rho \bar{u}^2$$

amongst them Sutcliffe (1936), and recently, Sawyer (1959).

Equating the expression for τ/ρ from Eq.2(1) and 2(2) we have

$$\sqrt{\frac{2}{C_D}} = \frac{1}{k} \ln. \frac{z}{z_0} \qquad 2(3)$$

showing C_D to be dependent on the reference height z, but not on the velocity at that height.

It is also convenient to introduce a subsidiary reference velocity $u_{(*)}$, defined by

$$u_{(*)}^2 = \tau/\rho \qquad 2(4)$$

leading to the relationship

$$C_D = 2\left(u_*^2/\bar{u}^2\right) \qquad 2(5)$$

Over ordinary agricultural land C_D is of the order 10^{-2} to 10^{-3} for a reference height of a few metres, and, in spite of the very great difference in scale, is of the same general order of magnitude as the coefficients of skin friction, at high Reynold's numbers, for artificial surfaces such as flat plates, wings, and struts. A useful list of values of C_D for surfaces of meteorological interest, reference height 2m, is given by Deacon (1953, p.19).

Of interest to the present investigation are comments by Sutcliffe (1936) on results he obtained on the frictional drag of ordinary countryside. With reference height 10 m. his value for C_D was about 0.01. Noting that the ratio of this value of C_D to that for a bluff body was of order 1 to 100, he pointed out that the frictional drag of a tree presenting a 10 m. square to the wind would be equivalent to that exerted by an area of turf 100 m. square, and suggested that level country broken by only one tree every hundred metres would be considered as "reasonably open". Making the convenient assumption that the horizontal projection of the tree canopy would also be a 10 m. square then some one percent of the area of "reasonably open" countryside would be occupied by trees. It is of interest to note that a one percent cover by an assemblage of bluff bodies would appear to be quite insufficient to reduce wind stresses on certain erodible surfaces to below the danger point, e.g. in the rather windy climate of Denmark tree plantations, which in 1929 covered 16.7 percent of Bornholm and 5.7 percent of South Jutland, were considered to be quite inadequate to combat the danger of fertile land being submerged by drifting sand, Andersen (1943).

Further reference to the variation of frictional drag with differing land surfaces is given in Chapter 4 on page 48.

An investigation of particular interest from the biological point of view has been carried out by Whitehead (1951, 1957). A study was made of the characteristics of some plant communities about 2800 metres in the Central Appenines. In particular, the mean heights of plant, and of inflorescence, were noted in four separate communities, and also for the same species in each of the four different areas. On a number of occasions wind speeds were measured at the four sites at various heights up to 1 metre with a multiple pitot-tube anemometer. A plot of log (height) against speed gave different, but consistent, values of z_0 for each of the four areas. In view of the extreme speeds sometimes experienced in this mountainous region, and the possible decisive effect of such extreme winds on plant growth, Whitehead postulated a speed of 90 miles per hour at 1 metre, and adopting the previously obtained values of z_0 , found that the mean heights of the four species, and more significant, those of the same species in the four different areas, all occurred at a level where the same absolute wind speed (c. 22.2 m. per sec.) was experienced. A tentative conclusion from this investigation is that, given a particular wind profile, the aerial portions of vegetation downwind will not penetrate into regions where the wind speed exceeds some limiting value. Whitehead here relates plant form to an extreme wind and not to a mean wind as was done by Putnam (1948), although of course numerical relations between mean and extreme winds may render either an index of the other.

Further relationships between the form of vegetation and wind speed are considered in Chapter 4, Section 2.

Section 2. Extension to very rough surfaces

With an assemblage of obstructions more than a few centimetres in height, it is clear that there must be more than one field of flow. With a complex surface such as a stand of wheat and even more markedly, a stand of trees, there are at least two different fields of flow. Deacon (1953) states the position in these terms: "A rough surface is essentially a three-dimensional assemblage of bluff bodies, and the mean effective level at which the drag is generated must depend upon the vertical distribution of the active surface and also on the closeness of packing of the roughness elements. The effective datum level for the turbulent flow must therefore be at some height between the base and the top of the roughness element, which can only be determined from the wind profiles".

If d be the height of the datum level above the ground surface, Eq. 2(1) is modified to read

$$\bar{u}(z) = \frac{1}{k}\sqrt{\frac{\tau}{\rho}}\ \ln\frac{z-d}{z_0} \qquad z > z_0 + d \qquad 2(6)$$

Earlier workers estimated d on general grounds, and more recently Jensen (1954) has found the same procedure satisfactory for the type of field work in which he was concerned. For more precise studies, Pasquill (1949), Deacon (1953), and Rider (1954), amongst others, estimated z_0 and d independently from observations of mean winds at several heights. These workers have found, however, that as the wind itself may alter the physical form of the surface, a variation in z_0 and/or d due to wind velocity is possible even in adiabatic conditions. Deacon, for example, reports that for long grass z_0 decreased with increase of wind speed, and this was ascribed to the bending over of the grass stems and the production of a flatter and smoother surface; no appreciable variation in d was, however, required.

Decisions regarding the position of the effective surface of a physically irregular one, arise also when dealing with temperature and humidity profiles. Pasquill in a Meteorological Office Research Paper (1947) concluded, from observations of wind temperature and moisture profiles over pasture (grass up to about 10 cm.), that "to a good approximation d is the same for momentum, temperature and absolute humidity and is unaffected by substantial variations in the quantitative criterion of atmospheric stability" (viz. an approximate form of the Richardson number). Long (1958), in an attempt to estimate the rate of evaporation of water from the leaf surfaces of a potato crop by a simple adaptation of a standard formula given by Rider (1954), cites unpublished justification for locating d at the level of zero vapour pressure gradient in the early morning. Geiger (1950, Chap.7) produces some pertinent observations regarding the effective surface for radiant exchange in irregular crops, and it is evident that the datum level for radiant exchange within a stand of vegetation will vary with the ease of penetration of direct solar radiation, and thus with solar altitude: for some recent data and some further references, Waterhouse (1955) may be consulted.

The extension to non-adiabatic conditions involves either the retention of the logarithmic profile, but a variation in z_o , or a constant z_o and a more generalised profile law which, in conditions of neutral stability, will degenerate into the logarithmic law. Deacon, who clearly stated the alternatives, first proposed the now generally accepted second method of dealing with the situation.

The variation of z_o , and often d, with the changing physical characteristics of a wind deformable surface will obviously render more difficult attempts to extend the treatment to forest canopies and similar types of obstruction.

Only a very few observations of wind speed profiles over forest canopies and similar surfaces appear in the literature. Fons (1950), Gisborne (1941), Kepner, Boulter and Brooks (1942), published results obtained under varying conditions, viz. those of Fons around midday in California during July, August and September although "collection of data was avoided on obviously gusty days"; those of Gisborne during the summer fire-risk period in Montana for 1938, 1939 and 1940, and those of Kepner et al. at night with appreciable temperature inversions from 3 degrees to 20 degrees F over the height interval 5 ft. to 60 ft.

Kepner et al. attempted to fit a power law profile to the observations above the canopy formed by trees 11 ft. in diameter, 11 to 12 ft. high, so spaced as to cover about 20% of the ground. A height z was measured from the ground and the profile law expressed as

$$\bar{u}(z) = c(z - z_r')^m$$

c, m and z_r' being adjustable constants as between sets of observations grouped according to the speed at 50 ft. The term z_r' was adjusted to give the best fit and was found to vary appreciably but not systematically, and a mean value of 10 ft. was adopted, giving rise to a "roughness parameter" of 1 to 2 ft. The logarithmic approach was sketched but no parameters evaluated.

Kittredge (1948) fitted a power law to the data by Fons, and Poppendiek (1949) a logarithmic law to all three sets of data.

The attempt by this last named writer is of some interest in the present context. He recognised two fields of flow, one above and one below z = h, where h is derived from the general description of the stand. Assuming a linearly increasing shear from $0 < z < h$ and a constant shear τ_1 above the canopy he derived a relationship

$$\bar{u}(z) = \bar{u}(h) + u'_{(0)} \ln \frac{z + z_1}{h + z_1} \qquad z > h \qquad 2(7)$$

$$u'_{(0)} \equiv \frac{1}{M} \cdot \sqrt{\frac{\tau_1}{\rho}} \quad \text{where} \quad l = M(z + z_1)$$

l being a mixing length appropriate to conditions in the air layer above the canopy. The apparent similarity between Eq. 2(7) and that derived by Bagnold (1954) for mean wind speeds over a surface of drifting sand is suggestive, although Poppendiek found that $\bar{u}(h)$ varied with wind speed, and from an analysis of the nocturnal wind data of Kepner et al., found that z_1 as well as τ_1 varied between particular groups of wind speeds.

were The data ~~was~~ not sufficient in quantity, and probably too heterogeneous to allow the derivation of more than very approximate values of the parameters, but he found that z increased from about -10 feet to zero as $\bar{u}(h)$ varied from 0.5 feet/sec. to 3 feet/sec.

are Only Gisborne's data ~~is~~ such as to enable a u/log z plot of wind speeds above a canopy to be made in what may tentatively be assumed approximate adiabatic conditions. Mean speeds were given for 2 hr. periods for each of the months of June, July and August in 1938, 1939 and 1940. In addition, data were given for the days in each of the months in which the highest and the lowest speeds were recorded. The mean height of the canopy was about 80 feet and wind speeds were measured at 2 ft. 49 ft. 83 ft. 112 ft. and 156 ft. Most of the observations are well represented by a relation of the form

$$\bar{u}(z) - \bar{u}(83) = A \log(z-80)$$

with A changing only slowly with u (156). Many of the deviations from linearity show some degree of convexity to the u - axis as is typical of lapse conditions.

have It would appear from such data as available that the pattern of air-flow over heavily vegetated surfaces is not inconsistent with a physical model similar to that adopted by Bagnold in his studies of wind speed over

2 a moving sand bed; the threshold velocity varying, however, with speed and associated with a lowering beneath the canopy level of the plane of separation between the general flow and that within the stand.

Chapter 3

The Determination of the "Zone of Influence" and "Efficiency" of a Wind Barrier

It is to be expected that many of the main features concerning the influence of obstructions on air flow will be most clearly revealed from studies in which the shape and other relevant physical properties of the obstruction can be readily defined. Such an obstruction is an infinite cross-wind barrier with a horizontal sharp edge, to which obstacles such as walls and fences approximate, and to which natural barriers such as hedges and tree belts correspond with varying degrees of accuracy. The extension of the study to barriers of considerable (downwind) width will be expected to introduce further complications and the final extrapolation to natural features such as ground contour can only be attempted with considerable caution.

Reference has been made to some of the voluminous literature on this topic and Caborn's recent treatise (1957) includes a wide ranging survey which it is not proposed to duplicate. There are, however, some topics of physical interest which will be examined a little further: these include the reconciliation of wind-tunnel and field results, the more precise specification of the area affected, and the influence of a series of barriers. The related topic of "topographical" shelter is more conveniently considered in Chapter 4.

Some confusion has arisen concerning two terms extensively used in shelter investigations. In the present work the following definitions will be adopted:-

"Zone of influence" - The distance downwind - occasionally the area in a vertical section, but more commonly in a horizontal plane - adjacent to a barrier in which some stated phenomenon, e.g. wind speed, heat loss, soil erodibility, is affected to some predetermined order of magnitude; expressed in terms of H or H^2 respectively (H = height of barrier).

"Efficiency" - The degree or intensity of the influence exerted, expressed either in relative or absolute terms; and either integrated over the whole "zone", or as experienced at particular points within the zone, or with respect to the maximum degree of influence exerted; (the most common source of confusion is the use of the term "efficiency" as an alternative to "zone" as defined above).

A very large amount of data from laboratory and field studies now available reveal that similarity of the two-dimensional flow patterns in the vertical (x-z plane), for barriers having the same geometric permeability (denoted by ϕ), has been largely achieved by expressing horizontal and vertical differences in terms of H and wind speed u at $z = z$ in terms of u_o (the incident wind speed). Ideally measurements either side of the barrier should be taken at the same distance above the ground, and such a procedure is usually adopted in wind-tunnel studies, but in the field it often happens that winds at several levels in the lee of the obstruction are expressed in terms of wind at some fixed height upwind. The variation of incident wind speed with height is small above a few feet from the surface, and in many practical cases involving hedges and tree belts (H ranging from 10 ft. to 30 ft. or more) errors are small when u is measured at $z > 3$ ft. above the surface. The wind aft of the barrier varies considerably with height above ground, but here again many practical needs are often satisfied by observations at some convenient height from 3 to 5 ft., provided this does not exceed about 0.4 H. These procedures become less satisfactory as ϕ

2

approaches zero.

Good qualitative agreement between flow patterns on all scales, particularly the identification of the eddy zone to the lee of all but the most open barrier, followed by a stream with increased turbulence, has been recognised now for some decades, but quantitative agreements have been approached only within the last ten or twenty years. Relative to many meteorological problems, scale effects are not great - a factor of 2 or so is involved - and the chief remaining differences are: the more extensive sheltered regions (expressed in terms of H or H^2), and generally the greater efficiency of a barrier, indicated in the laboratory as compared with that achieved under field conditions; and the various possible patterns of air-flow in the eddying region which different measuring techniques suggest.

Examination of published results indicate that discrepancies arise mainly from three interrelated sources:-

(i) Differences in instrumentation and measuring techniques;

(ii) Differences between the structure of the air-stream in the open and in the wind-tunnel;

(iii) Difficulties attached to the creation of a realistic model for many forms of natural obstruction, especially hedgerows and tree belts.

Considerable progress has been made in dealing with topics under (ii) in those meteorological conditions important in the context of the shelter-belt problem, but (i) and (iii) still appear to require more attention.

An important distinction between field and laboratory conditions, which has far reaching consequences, is the variation in the mean direction of the atmospheric wind and its characteristic gustiness. The resolution of the first difficulty will be partly effected through techniques of analysis which it is hoped will emerge from studies described in Part II, allied to work such as Jensen's on the variation, with wind direction, of the size of the area protected by a shelter system. The second difficulty, that relating to gustiness, may be susceptible to laboratory study if a pulsating flow is introduced. An excellent wind tunnel indication of the differences between a steady, though fully turbulent, flow and a pulsating one is given by Irminger and Noddentved (1936) in their investigations of flow around model buildings. When the air stream was suddenly stopped, the eddy on the lee edge of the building was rapidly replaced by one with an opposite rotation giving rise to very disturbed conditions. Hence we may infer that a natural wind is likely to lead to alternating build-up and collapse of flow pattern. The fluctuations in flow are more serious with an impermeable barrier than with a lattice or filter-type barrier, in which latter case the full scale phenomena are more in agreement than with those revealed in the laboratory, e.g. Hallberg (1943).

Section 1. On instrumentation and measuring techniques

In wind-tunnel work the pitot-tube assembly has been the customary instrument, whilst in field studies the cup anemometer has been the usual tool. Important exceptions are the investigations of Jensen(1954), who employed a sensitive swinging plate anemometer calibrated against a pitot-static tube, and studies by Woodruff and Zingg (1955), in which the last mentioned device was used in the field. On both scales the wake has been explored by streamers (e.g. Hallberg, 1943), by the deposition patterns of airborne or erodible materials, Finney (1934), and generally by smoke. The direction of flow has usually been inferred from smoke patterns etc., one of the few quantitative studies known to the writer being that of

Nageli (1953), who also recorded the mean direction as indicated by a number of small wind vanes disposed around reed fences.

As is now well established, the wake behind a relative dense barrier ($\phi < 60\%$ approx.) consists of a zone 10 H or 15 H wide, in which Long? eddies, having diameters of the order H, are the dominant feature, followed ? by a region in which it is reasonable to postulate a mean flow with a residual turbulence. In most laboratory studies no attempt has been made to assess the degree of air movement within the eddying region. As for many practical purposes, e.g. the increasing of crop yields, the protection of livestock and buildings, this eddying zone is the more important region, as much information on the flow therein is highly desirable. Some of its main features have been identified by smoke patterns, and Woodruff and Zingg (1952) established its floor level boundaries from the patterns assumed by an erodible sand.

Fig. 3(1) derived from Caborn's treatise clearly illustrates some of the points at issue.

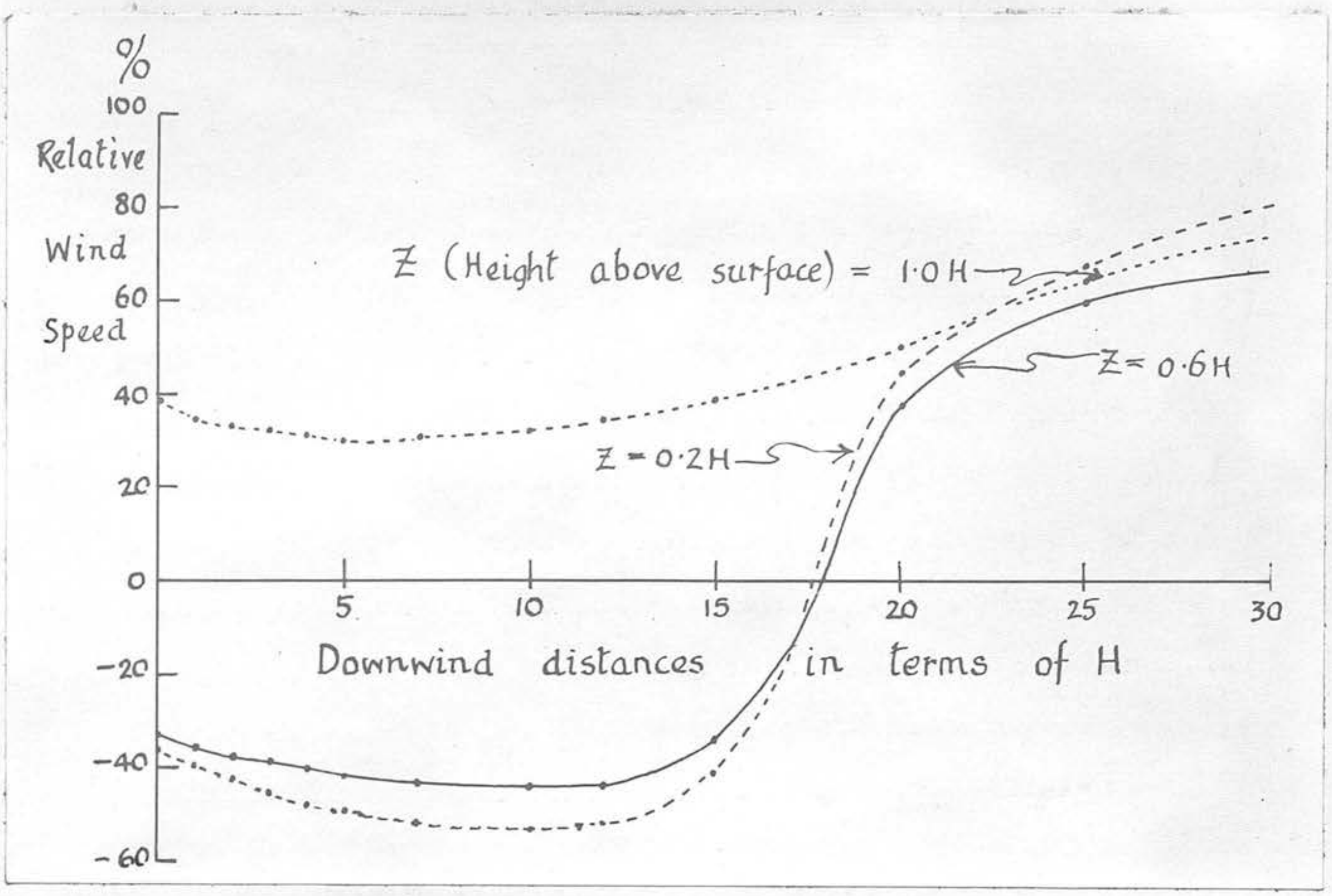


Figure 3(i). Variations in relative wind-speed to the lee of a dense barrier

(After Caborn (1957). Tables 4, 5, 6; Model Design 1C)

A pressure less than the static head in the undisturbed stream was recorded in the lee of virtually impermeable barriers for a distance downstream to 18 H by total pitot-heads facing upstream and placed at levels z = 0.2 H and z = 0.6 H. At levels of z = 1.0 H and above, the pressure exceeded the static pressure. The formal negative velocities indicated in the diagram correspond to a deficit below static pressure of about 25% of the dynamic head of the free stream, in agreement with Irminger and Noddentved (1936), who found a pressure deficit of this magnitude extending to at least 11 H downwind, and maintained over the face of a model building, height H, in the wake of a barrier of the same height. The interpretation of conditions in this 0 H to 18 H zone in terms relevant to most practical requirements, i.e. in terms of total displacement, is not clear. The implication of a considerable horizontal wind towards the barrier followed by a <u>drop to zero</u> and then a steady recovery to the incident velocity does not correspond to field experience. There is, of course, no doubt but that return currents do occur within the eddying wake up to a level of about $z \simeq 1$ H, e.g. from field observations on snow drifts, observations on smoke trails (Nageli, 1953), and many obtained under laboratory conditions, but the writer knows of no case of published field observations showing a zone of minimum air movement between this eddy region and that of the re-established flow. A "total" head equal to the static head in the undisturbed stream might be expected if the local current moved in a plane perpendicular to the total-head tube used by Caborn.

Such marked scale effects as mentioned above do not arise if the stream in the wind-tunnel is rendered fully turbulent (as ensured by Jensen, Woodruff and Zingg and others), nor when permeable screens are under investigation, but the difficulty of effecting a quantitative reconciliation remains.

It should be stated that these considerations do not invalidate Caborn's conclusions on the problem which he set himself, the data in Fig.3(1) having been reproduced merely to pose a number of questions in a rather striking fashion.

Woodruff and Zingg (1952) conducted laboratory studies using an erodible sand and found that in the zone between 3 H and 8 H behind an impermeable barrier, material of a particular grain size was swept away, revealing therefore an unprotected region in addition to that which occurs beyond the bounds of the zone of influence. In this eddying region "(flow) was very turbulent, moving in all directions in general and towards the obstacle in particular". Comparative data are given in Table 3(1), of the protected region defined in terms of wind reductions of 25% and 50% as revealed by the sand patterns, and as indicated by pitot-tube measurements at z = 0.1 H. It will be seen that there is little difference based on the criterion of a 25% reduction in wind speed but a significant difference if the permissible upper limit for wind speed is $<$ 50% of the incident value.

Table 3(1) Extent of protected zone behind barriers having various cross-sectional profiles, defined by reference to wind reductions of 25% and 50%, and as revealed by measurements with pitot-tubes, and by the boundaries of an erodible sand for which the threshold velocity was known (from Woodruff and Zingg, 1952)

Type of Obstacle	Zone as indicated by: Pitot-tube measurements at z = 0·1H		Zone as indicated by: Sand boundaries	
	50% reduction	25% reduction	50% reduction	25% reduction
Vertical Plate	15.5 H	21.5 H	10.2 H	23.5 H
Plate of Triangular Section	15.0 H	20.5 H	9.5 H	22.5 H
Cylinder	9.0 H	14.0 H	6.0 H	20.5 H
Model trees	13.5 H	27.0 H	7.5 H	25.0 H

Although the response characteristics of many devices for measuring wind speed are now well established, no comparative study using a range of instruments in the eddying zone behind an impermeable barrier in the open has at any rate been widely published, and this renders it difficult to decide to what extent instruments and techniques on the one hand, and air-flow characteristics on the other, contribute to differences between field and laboratory results.

In conditions where it is justified to assume a mean flow with a residual turbulence, both the pitot-tube and the modern cup anemometer give a close approximation to the true mean air speed; both instruments are relatively insensitive to angles of yaw less than about 30°, and in a pulsating flow both instruments over-read. Beyond about 10 H from a dense barrier and 3 H or so from a permeable one ($\phi > 40\%$) the flow is such that observations of air speed derived from most types of wind recording instruments will be closely similar. Within the eddying zone, however, flow conditions are such that instruments will respond differently, and methods for the ready conversion of one set of readings to another are not yet available. It seems unlikely that exact conversion factors can ever be derived, but approximate factors - adequate for a particular purpose - might be expected.

The specification of permeability.

Almost invariably the permeability (ϕ) of a barrier is merely defined in terms of the ratio of the total unblocked area to the total area enclosed by the external boundaries of the vertical projection of a barrier; ϕ should thus be regarded as the "geometrical" permeability. A given value of ϕ may be associated with an infinite variety of possible patterns of blockage elements and of materials. It is true that attention has been given to the coarser features of the pattern, e.g. whether more open at the top than the bottom, whether the blockage elements are basically horizontal or vertical etc., but there seems doubt if the possible influence of mesh size and material on the performance of a lattice type of obstruction has been sufficiently investigated.

The resistance offered by the fences, walls, etc. to air-flow is usually due to form drag, but clearly the mesh size and surface properties of the materials used, especially in model studies, may well reach dimensions in which the contribution of viscous drag to the total resistance would need to be considered. It also seems reasonable to suggest that some of the resistance offered to an air stream percolating through a dense hedge or a wide tree belt will arise from viscous drag. As will be seen later, whilst good quantitative agreement has now been achieved between wind-tunnel and field studies using artificial materials, discrepancies have arisen when comparing results obtained with tree belts and what are regarded as suitable models of equal permeability.

Kreutz and Walter (1956) carried out laboratory investigations with mesh screens composed of various materials. They found that the protective effects of cotton screens with meshes 2.5 mm and 6 mm respectively were greater than with wire meshes of 1.5 mm and 2.5 mm, although the contrast diminished as the mesh size increased. Cocoanut fibre matting with a mesh of 25 mm was found to give substantially greater wind protection to 10 H than metal and wooden lath barriers of equal geometrical permeability. Results of a similar nature were obtained by Noddentved (1938 (a) (b)), who observed that fences of rough surfaced materials, e.g. ropes, straw, were more effective in reducing the wind than rigid smooth-surfaced materials.

The data of Kreutz and Walter suggest that when the mesh dimension exceeds a value somewhat greater than 6 mm it ceases to be of major importance, but that below that figure the effective permeability becomes progressively less than the geometrical permeability. Variation of surface roughness between different materials seems to be significant up to a mesh size of about 25 mm, although it may be conjectured that surface roughness is unimportant with very fine meshes.

In laboratory work, model screens are typically a few centimetres high only and the spacing between blockage elements of the order of 1 cm or rather less, and it is consistent with the evidence above that if rigid smooth-surfaced materials are used, direct comparisons between results using model and actual fences is possible.

Section 2. The reconciliation of wind-tunnel and field measurements of air-flow behind a barrier

In their classic study of the effects of the wind on buildings and other structures, Irminger and Noddentved (1936) observed that if a single vertical barrier with a horizontal sharp edge is placed on a surface along which a steady two-dimensional flow is established, the surface boundary layer will leave the "ground" some appreciable distance upstream. Almost invariably in the open the boundary layer is turbulent and, on approaching the obstruction, will grow vertically, the upper boundary layer following an easy curve (convex to the ground) from a point a few multiples of H upwind of the base of the barrier to its upper edge, cutting off a region to windward in which eddies are present. From the upper edge of the barrier the surface of separation continues to rise and a turbulent wake is produced to the lee of the obstacle. The existence of the windward vortex prevents the establishment of the full stagnation pressure on the upwind face, and from our point of view the important contribution by these investigators lay in the attention they drew to the influence of upwind and downwind ground roughness on the flow pattern, and in particular, the effect of upwind roughness on the development of the windward vortex region. They experimented with various degrees of roughness obtained by sanding the tunnel floor, by using corrugated paper and similar materials, and specified the effect on flow by means of a characteristic depth δ defined by

$$\frac{1}{2}\delta u_{(0)} = \int_0^\infty (u_0 - u)\, dz$$

Similarity of flow in various situations was assured by adopting the same value of the ratio δ/H. Their experimental range for δ/H was from 0 (completely smooth surface) to 0.27, the most usual value being 0.094, and, as was expected, the extent of the vortex region increased with increasing friction, i.e. with δ/H.

With a square screen, in contrast to the infinite cross-wind barrier, the upwind vortex region was more limited and the need for agreement in δ/H between scales of less importance.

Subsequent workers, of whom Jensen (1954), Woodruff and Zingg (1952) may be cited here, paid special attention to the need for a fully developed turbulent flow in wind-tunnel studies. Work reported by Woodruff et al. in 1952 was mainly concerned with obtaining, in the laboratory, reproducible results of a type agreeing with the "reported field measurements". No formal comparison was made in that paper with any particular set of field observations, but good agreement with such observations of that type was claimed, including the reproduction of certain important "second order" effects - amongst these, the smaller angle of upward divergence of flow when leaving a belt of trees, actual and simulated, compared with that obtained with a solid sharp-edged barrier, and the more extended wake for a dense tree barrier than for a solid object. Similarity of flow was produced by roughening the tunnel floor with gravel, grain diameter one-quarter to one-sixth of an inch resulting in a turbulent boundary layer "several inches deep at the 42 ft. point". As the models were 4 inches high and as the first measuring station was only 40 ft. from the tunnel entrance, some doubt must remain as to the validity of observations at levels of z = 3 H and above.

They established that, for air speeds of 24.8 ft./sec. 37.1 ft./sec. 43.4 ft./sec., the flow patterns behind a flat plate were almost identical. The possible sensitivity to the Reynolds number when scaling-up results was admitted, although by choosing a relatively high value, (viz. $\geqslant 5 \times 10^4$ in the wind tunnel; for 30 ft. high trees Re. $\simeq 7 \times 10^6$), it was claimed that the flow patterns should be relatively insensitive to this parameter: the possibility that Reynolds number may not be the most appropriate measure for atmospheric studies was also recognised.

Jensen compared the flow around model and field scale slatted fences, and achieved similarity by ensuring that

$$\frac{z_0\,(\text{model})}{z_0\,(\text{field})} \simeq \frac{\text{model barrier height}}{\text{actual barrier height}}$$

In formulating the profile laws for field conditions he estimated (where necessary) the appropriate zero-plane displacement d (usually the average height of plants in a fairly uniform stand).

Amongst a number of observational series illustrating the success of this method, one series will serve as illustration. Values of the relative wind speed, i.e. $\bar{u}/\bar{u}_0$ have been read from one of Jensen's graphs (Fig. 176, page 176) for screens of permeability 38% and at measuring height z = 0.4 H.

Table 3(2). Relative Wind Speeds (in %) for Permeable Screens (ϕ = 38%)
(i) in a smooth wind tunnel
(ii) in a roughened wind tunnel
(iii) in the open
(After Jensen, 1954)

Test Conditions	Distances in terms of H													
	4	6	8	10	12	14	16	18	20	22	24	26	28	30
	Percentage of "free" wind													
(i)	10	15	20	26	32	37	42	47	52	57	63	68	73	78
(ii)	15	25	34	42	51	60	67	72	78	82	85	88	90	92
(iii)	22	27	31	37	44	51	59	67	75	82	86	90	92	94

He was, however, unable to obtain close agreement between model screens on the one hand and hedgerow and tree belts of presumed equivalent permeability on the other, the sheltering effect of the latter being less than that of the corresponding artificial barrier. Part of the difficulty may be ascribed to the assignment of a correct (geometrical) permeability (ϕ) to a natural barrier, but an inspection of his results on pp. 180-184 suggests that this cannot adequately account for the difference.

A further comparison is that between some of Jensen's observations and some from the extensive field work reported by Nageli in the latter's 1953 paper. During the five winters 1948/1949 to 1952/1953 Nageli made observations of the flow as affected by reed fences 2.2 m. high of permeabilities 15 - 25% ("dense") and 45 - 55% ("open" barrier). In Table 3(3) relative wind speeds obtained by Jensen in a roughened wind tunnel with a screen of ϕ = 51%, and by Nageli for the "open" barrier are given.

Table 3(3). Relative Wind Speeds (in %) for Screens of Comparable Permeabilities under Different Conditions
(i) roughened wind tunnel at z = 0.4 H ϕ = 51% Jensen (1954)
(ii) field conditions, z = 0.5 H reed fences, ϕ = 45-55%. Nageli (1953)

Test Conditions	Distances in terms of H												
	2	4	6	8	10	12	14	16	18	20	22	26	30
	Percentage of "free" wind												
(i)	15	25	34	43	51	58	64	69	74	78	80	87	92
(ii)	43	46	50	54	57	63	69	75	80	84	87	90	93

Assuming, as seems reasonable from the published accounts, that the natural roughness at Nageli's site is equivalent to that adopted by Jensen in the laboratory, the agreement beyond 10 H is reasonably good between results for these two artificial barriers whose only stated common feature is approximate equality in ϕ. For the range 0 H - 10 H, wind tunnel results indicate a considerably greater degree of shelter, but some of this discrepancy is almost certainly due to instrumental differences (Nageli employed cup anemometers).

Broad similarity in the vertical (xz) and horizontal (xy) planes has been achieved as between fences in natural conditions and models in turbulent streams. With respect to the xz-plane, obvious practical difficulties doubtless explain the lack of appropriate data relating to full scale tree belts.

Some pertinent results are summarised in Fig.3(2).

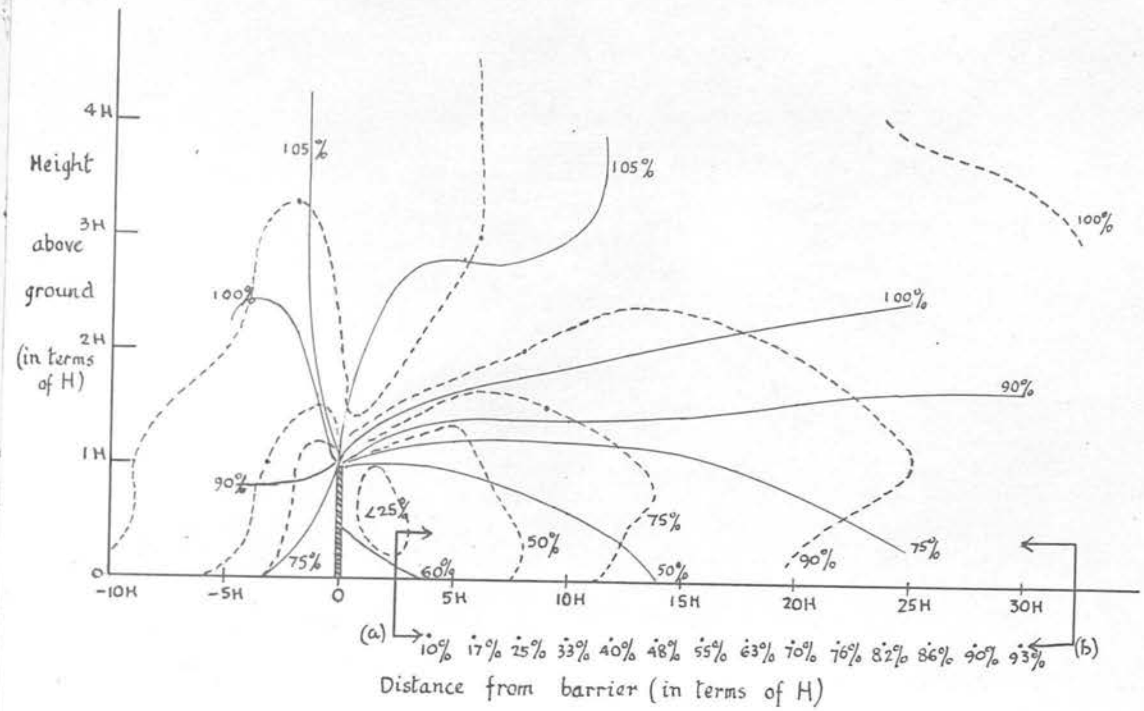


Figure 3(2). Values of Relative Wind Speed (%) as determined by

(i) Woodruff and Zingg (1952). Model tree belt; turbulent flow

(ii) Nageli (1953); Field measurements. Reed fences ϕ = 15-25%

(iii) Jensen (1954). Model fences Rough tunnel. ϕ = 30% (a)- - - -(b)

Amongst the features shown in the diagram are the somewhat greater wind speed reductions above z = 1.5 H in the zone from about 5 H - 15 H for the field barrier as compared with the model tree belt:- the formation and breakaway of large eddies from the ridge of the barrier in natural winds might explain some of the discrepancy.

It may be claimed, therefore, that insofar as unidirectional steady flow is concerned, the introduction of an appropriate surface roughness in the wind tunnel has considerably reduced the discrepancies between field and laboratory results evident in the earlier work.

Section 3. The restoration of the velocity field behind an obstruction

Three interrelated, but separately measurable, criteria suggest themselves, viz.:

(i) The re-establishment of the vertical profile of mean wind speed (in the present context, the logarithmic profile);

(ii) The re-establishment of the directional field;

(iii) The re-establishment of the smaller scale fluctuations as expressed by some form of gustiness factor.

(i) The re-establishment of the logarithmic profile.

Tanaka and Tanizawa (1953) have examined the flow at several levels to 2 m., downwind of a small hedge of height 80 cm. and crosswind length 15 m. They found the incident flow reappearing, at any rate to 2 m., at downwind distances which varied with the permeability,

viz.	ϕ = 80%	at 1 H
	ϕ = 50%	first at H but not finally until 15 H
	ϕ = 30%	at 10 H
	ϕ = 0%	at 15 H

From Nageli's (1953) investigation vertical profiles behind reed fences 2.2 m. high may be derived. In Fig.3(3) profiles at - 5 H and - 10 H (i.e. upwind) and at multiples of 5 H downwind to 30 H are plotted.

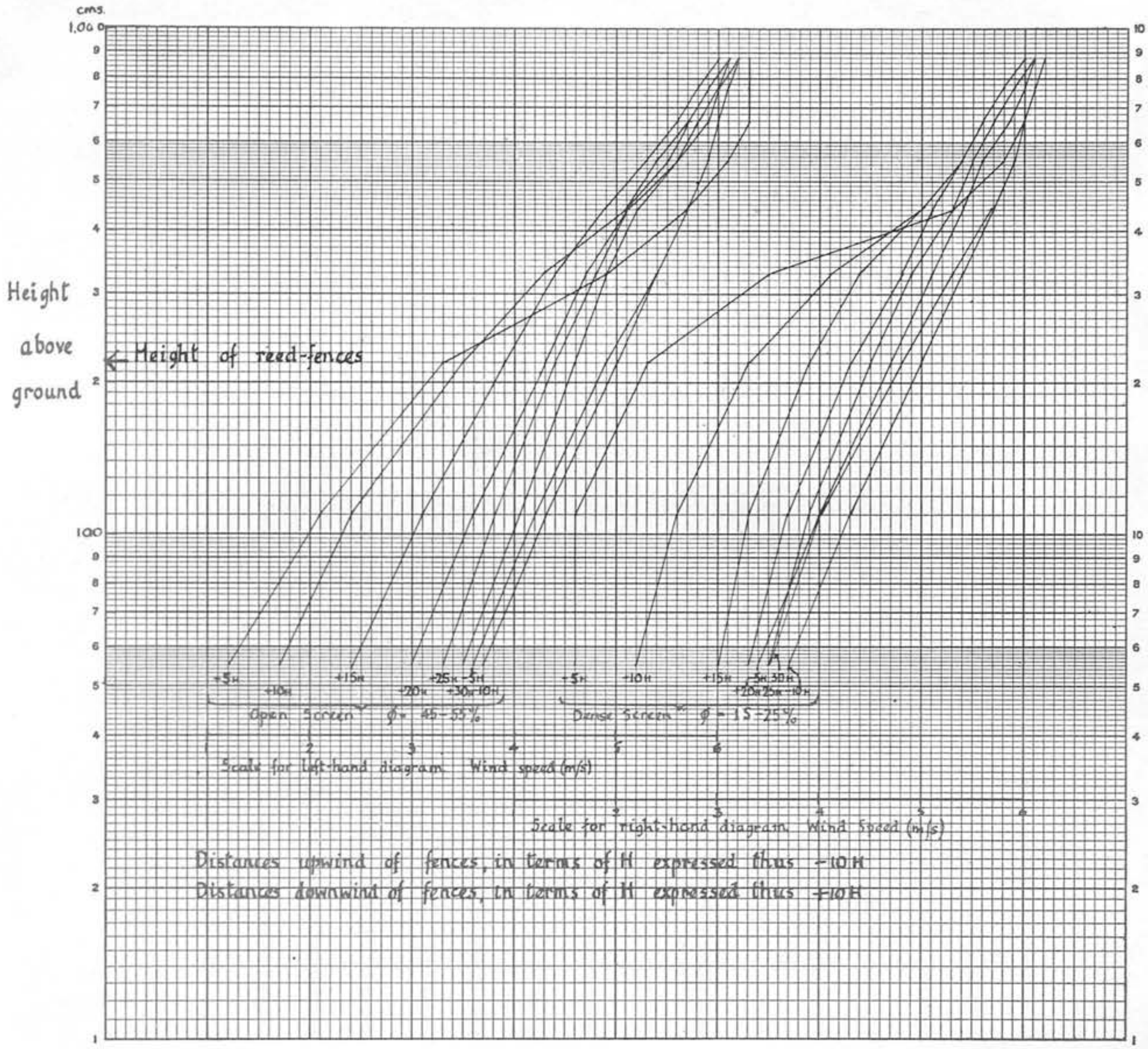


Figure 3(3). Vertical Wind Speed Profiles at Various Distances Upwind and Downwind from Reed Fences of Permeabilities ϕ = 45-55% and ϕ = 15-25%. Free wind speed 5 m/s at height 2.2 metres.

(From Nageli 1953)

The ground cover was uniform with grass some 10 cm. high: no zero plane displacement has been adopted, and a u/log z plot gives the acceptable result of $z_0 \simeq 2$ cm. For both types of screen a tolerably satisfactory logarithmic profile reappears at about 20 H, but in neither case is the incident profile established by 30 H downwind. Above a level 3 H from the surface there are differences which are doubtless of physical significance but probably of little practical importance to many aspects of shelter policy, and in the case of the denser screen these remarks might well apply to levels above 2 H. Below $1\frac{1}{2}$ H and in the zone 0 H - 20 H there are large divergences from the logarithmic profile in both cases and these are obviously more marked near the denser of the two screens.

The restoration of incident conditions to the lee of an extended canopy and of a succession of simple barriers is mentioned later (p. 42), but it may be stated that evidence is scanty and somewhat conflicting, arguments having been advanced suggesting zones of both greater and smaller extent than that for the single barrier of equivalent permeability.

(ii) The restoration of the directional field.

Apart from qualitative inferences from streamers and smokes there is little published information to consider, except for some field data given by Nageli, notably in his 1953 paper. From a large number of "spot" observations of the wind direction as observed from bi-vanes, the following conclusions may be drawn from Figure 9 in Nageli's paper:-

Changes in direction (observed at $z = H/4$) from incident conditions are evident at distances 5 H upwind of both "dense" (ϕ = 15-25%) and "open" (ϕ = 45-55%) screens. At about 2.5 H to the lee of the open screen the incident mean direction is broadly re-established, although the range is greater up to at least 15.5 H. With the denser screen the re-establishment of the mean direction does not occur before at least 5 H is reached (in all probability not before 10 or 15 H). In the lee of a dense screen variations in direction are very marked and at 3.5 H there is a reverse current at level H/4 (smoke trails also reveal a return current at $z = H$).

The directional distribution of winds at z = 0.7 H, in the free stream and at 4 H to the lee of the dense screen, is given in Table 3(4).

Table 3(4). Percentage Distribution of Wind Directions for the Free Wind, and at a Point Distance 4H Downwind of a Dense Screen, measured at Height 0.7 H above Ground Level

(from Nageli, 1953)

	SSE+ S.	SSW+ SW.	WSW+ W.	WNW+ NW.	NNW+ N.	NNE+ NE.	ENE+ E.	ESE+ SE.
	(Percentage)							
Free Wind (a)	0	2	38	49	9	1	1	0
In lee of dense barrier (ϕ = 15-25%)	10	6	12	14	7	6	17	28

(a) the free wind is strongly canalised by topography in the experimental area.

Information is also given of the inclination of the wind to the horizontal at the same two observing stations.

Table 3(5). Percentage Distribution of Wind Direction in a Vertical Plane
$z = 0.7$ H
(from Nageli, 1953)

Angle from Horizontal (Counter- clockwise positive)	Free wind	In lee of Barrier (ϕ = 15-25%) at 4 H
+85° to +90°	1	9
+51° to +85°	1	6
+17° to +51°	9	16
+17° to -17°	68	17
-17° to -51°	17	20
-51° to -85°	2	3
-85° to -90°	1	10
"Turbulent"	1	19

It is therefore quite evident that in the lee of a dense screen, certainly to 4 H, probably to about 15 H, the physical model of a mean flow with a superimposed "small scale" turbulence is invalid. It follows that the pressure measurements obtained from a pitot head directed towards the barrier cannot readily be converted to displacement (although the possibility of establishing useful empirical comparisons in specified conditions with the two instruments running in parallel still remains). The indications from Nageli's work are that with the more open type of barrier the conversion from pitot tube measurements to displacement may be a rational procedure.

(iii) The re-establishment of the free-stream turbulence.

Field measurements of the turbulence behind barriers on a scale now conventionally termed "intermediate" (i.e., sampling periods of from 2 - 15 minutes) have been reported by several workers.

Woodruff et al. (1955) examined the turbulence aft of a 4 ft. high slat snow fence of ϕ = 60%. Readings were taken every 30 seconds in ten minute runs from pitot-tubes at upwind, and at various downwind, stations. Gustiness was expressed in terms of $\sigma / \bar{u}$ where σ was the root-mean-square value of the twentyone observations of instantaneous speeds derived from pressure differences within the sampling period. In the free wind, at least up to a height of 6 ft., they found $\sigma / \bar{u} \simeq 0.15$, agreeing well with the result given by Deacon (1953, page 4) who used a sensitive pressure plate instrument for determining "instantaneous" values. The observations of Woodruff are mainly published in the form of rather small scale graphs from which the following table is derived.

Table 3(6). Values of Fluctuation Ratio $\sigma/\bar{u}$ in the Free Wind (i.e. at -10 H) and at Various Points in the wake of a Slatted Snow Fence H = 4 ft. ϕ = 60%
(After Woodruff and Zingg, 1955)

Level of Measurement in terms of H	Station			
	Upwind		Downwind	
	-10 H	+2 H	+6 H	+12 H
$\frac{1}{8}$ H	0.15	0.40	0.58	0.32
$\frac{1}{4}$ H	0.13	0.22	0.30	0.28
$\frac{3}{8}$ H	0.15	0.18	0.20	0.25
$\frac{1}{2}$ H	0.15	0.18	0.17	0.23
$\frac{3}{4}$ H	0.15	0.22	0.19	0.23
H	0.13	0.23	0.18	0.20

The permeability is high (ϕ = 60%), and the inference from Nageli's observations when ϕ = 45-55% (see p. 37) would indicate basic drift at 6 H and beyond in the direction of the incident stream, but it is clear that free stream turbulence had not been reached by 12 H.

Tanaka and Tanizawa (1953) examined the turbulence induced by small wooden barriers H = 80 cm. and of length 15 metres of differing ϕ. As far as can be judged from the original paper, turbulence was specified by $|\overline{u'}|/\bar{u}$ where u' is the deviation of the instantaneous value from $\bar{u}$. Results are given in Table 3(7).

Table 3(7). Values of "Gustiness" upwind and downwind of Wooden Barriers of Different Permeabilities (After Tanaka and Tanizawa, 1953)

Distance from barrier	Gustiness ratio			
	Permeability (%)			
	0	20	50	80
-5 H	0.27	0.28	0.25	0.24
-3 H	0.28	0.29	0.27	0.26
- H	0.48	0.35	0.32	0.27
+ H	0.50	0.86	0.38	0.13
+3 H	0.93	0.80	0.47	0.34
+5 H	0.46	0.42	0.36	0.32
+10 H	0.36	0.34	0.33	0.31
+15 H	0.34	0.30	0.27	0.28
+20 H	0.29	0.27	0.24	0.25

Although the absolute values differ somewhat from those in Table 3(6) the general pattern is broadly concordant.

Nageli (1953) gives observations of "sharp" movement of the wind vane with which he associated the passage of eddies. "Fluctuations" occurred much more frequently behind the "dense" than aft of the more "open" of the reed fences, and were observed nearer the former (viz. at 3.5 H) than the latter (viz. at 5 H).

The evidence presented in this section may be summed up as follows:

By about 20 H from a single barrier the $u/\log z$ profile has reverted to approximate linearity, although shifted bodily towards rather lower speeds (this is characteristic of a change to increased surface roughness), but co-incidence with the original profile may be delayed until after 30 H.

Within fairly broad limits both the directional and the eddy distribution of the original stream are re-established by about 20 H.

Section 4. The "zone of influence" and "efficiency" of a succession of barriers

Caborn (1957, p.17) has summed up a considerable amount of work on possible cumulative effects of a series of parallel shelter-belts. He concludes that with belts more than 6 H apart, no cumulative shelter effect is apparent downwind of the most lee-ward belt, although he still accepts earlier data of Bates (1945) that a system of four parallel belts separated respectively by distances of 25 H, 20 H and 30 H created a "larger coherent mass of stilled air" with a zone "7 to 12 H stretching laterally from the ends giving some degree of protection". Jensen (1954) demonstrated that if two barriers were placed more than 5 H from each other there was little difference in shelter effect downwind of the second barrier from that produced by a single barrier of density equivalent to that of the double barrier. With a 2 H interval between barriers there was a more marked effect leeward of the second up to 20 H but smaller beyond that - the type of result obtained when the density of a single barrier is increased. Field work on the whole confirms no marked cumulative influence of barriers greater than 30 H apart, although Nageli (1946) and Edlin (1953) suggest that with barriers 28 H or less apart, the general wind reduction in the intervening areas is greater than in the absence of the successive barriers. The apparent absence of marked reverse flow within systems of belts is a point in their favour in spite of no net increase in the zone of influence.

Woodruff (1954), Woodruff and Zingg (1955), Woodruff (1956), report a series of investigations with models of fences and tree belts and with full scale fences in which the effects of a succession of obstacles is considered. Spacing intervals between successive barriers varied from about 3 H to 24 H in the several investigations. Broadly their findings are that shelter effects aft of the last barrier of a series are not greater than for a single barrier (at least if the series is of four or fewer elements) although between barriers there is an enhanced effect. For more than four barriers they regard the question of cumulative effect as still open, based upon the finding (1955) that the isopleths of relative velocity continue to rise in the interval from the first to the last obstacle, which "would indicate an adjustment towards a lower velocity above the fence with distance. A continuation of this trend over a larger number of successive fences would probably mean that the effect would extend down to the height interval between the fences, thus giving an accumulative effect from the use of successive barriers ....previous studies of air flow about shelter belts or other barriers to the wind flow have not included a sufficient number to substantiate .... the conclusion (i.e. no cumulative effect can be obtained). It appears reasonable, however, when the fences are considered to be roughness elements of great magnitude affecting the wind flow as do smaller surface roughness elements, that the separation boundary is moved up an appreciable distance above the ground leaving a quieted zone in the roughness wake".

In the limiting case, parallel belts converge to form an uninterrupted obstacle, such as a forest, and current opinion on the basis of wind tunnel work by Blenk (1952) and Caborn (1957), and field work by the last named authors and by Nageli (1946, 1953), is that when a critical downwind width of belt has been achieved (5 H in the wind tunnel) subsequent increases are accompanied by a decrease in the zone of influence expressed by the phrase that "a wind belt consumes its own shelter". These latter results are
2 contrary to some admittedly earlier field work by Noddentved (1940) who found that to the lee of a plantation extending downwind for more than 2000 m., the zone of influence was some 60 H - 70 H compared with 30 H - 40 H for plantations of width less than 2000 m. Noddentved suggested this was due in effect to the complete establishment of a new boundary layer extending to heights greater than that in the case of a low crop cover or of the disturbance to flow caused by a narrow barrier: Jensen (1954) finds some support for this in wind tunnel investigations, which indicated a greater sheltered zone for wide models (width/height ratios > 60) than with narrower ones. The lengthy quotation from Woodruff et al. in the previous paragraph also indicates the possibility that wide barriers may be more effective than narrow ones.

Section 5. The numerical specification of the sheltered area.

In its most general form the protected zone is a three-dimensional one bounded by the ground, the barrier, and some selected surface over which the relative wind speed is constant. Of the various attempts devised for computing a shelter effect in practical cases, reference will be made only to two having some physical interest. Frankenberger (1952), in an Appendix to a paper, gave an analytical description of conditions in the vertical plane. The origin O (see Fig.3(4)) of his system of co-ordinates was placed at the top of the barrier with Ox horizontal and Oy vertically downwards where:

x = horizontal distance from barrier to measuring point in terms of H.

y = H - h where H = height of barrier.

h = height of measurement.

Isopleths of $\bar{u}/\bar{u}_{(0)}$ are given by

$$\bar{u}/\bar{u}_{(0)} = 1 - \exp\left(-\frac{1}{9}\frac{x}{y} + \frac{1}{3}\right)$$

and consist of straight "rays" emanating from O.

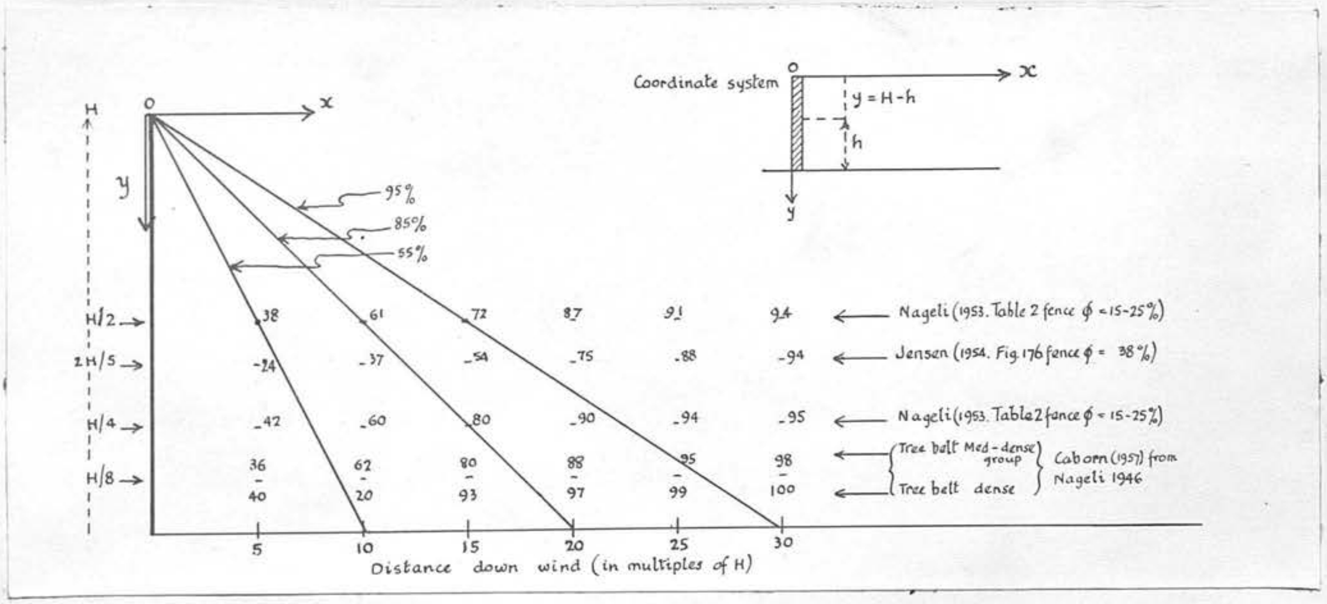


Figure 3(4). Comparison of isopleths of computed relative wind-speed (in per cent) after Frankenberger (1952), with some observed results as indicated.

Certain limitations are evident:-

if $h = H$ then $x/y = \infty$ and $\bar{u} = \bar{u}_{(o)}$

i.e. there is no provision for the rising surface of separation, and when $x/y < 3$, the expression has no physical significance.

In Fig.3(4) three isopleths (or "rays") have been drawn corresponding to relative wind speeds of 55%, 85%, 95% and, for comparison, "spot" values from several sources are inserted at the appropriate level. Frankenberger notes good agreement with some of Nageli's field results (1943, 1946) on tree belts - in these cases $H \simeq 15 - 30$ m. and the height of measurement was 1.4 m. Agreement is poor when $h > H/2$ and, as implied above, fails when $h = H$. The method, however, does draw attention to the importance of stating the height of measurement and would appear to be a useful aid for field work in which the flow around tree belts of varying heights from, say, 15 ft. upwards is being explored by anemometers at various levels, provided none of these exceed about $H/4$.

Woodruff (1956) examined in the wind tunnel the relationship between the area (in vertical section), within which the speed had been reduced to below stated percentages of the incident wind speeds, and the spacing interval between systems of fences of various structures. One example from the paper will serve to illustrate the results:

if S = spacing interval (in multiples of H), and

$I \equiv$ an "effectiveness index" (being the area, in terms of H^2 in which the wind speed reduction was to some stated level or below - 50% for the case in question)

it was found that

$$I = 4.78 + 0.21S - 2 \times 10^{-4} \exp (0.39S)$$

as S took various multiples of H on the range 8 to 30, implying a "critical maximum" for this particular system when the interval between successive belts is 20.1 H.

With respect to the horizontal (xy) zone at ground level due to a single obstruction, it is immediately evident that currents deflected around the ends will tend to meet again some distance aft of the barrier, and that such edge effects will be proportionately less important as the length of the barrier is increased. Woodruff et al. (1952) examined the shape of the ground level zone behind impermeable barriers of finite widths 6 H, 12 H, 18 H, 24 H, and 30 H (H = 1 inch) placed at 90°, 45° and 30° to the incident flow. They found that "the zone was parabolic in shape regardless of the angle of approach and proportional to the square of the length of the barrier normal to the wind for projected lengths up to 20 H". In appraising these results it must be remembered that the tunnel width was only 36 inches.

It may be remarked that measurements in the field with winds oblique to the barriers are insufficient in number and range to allow a coherent picture to be built up. In the main the general parabolic sheltered area for finite cross-wind barriers has been demonstrated, e.g. Jensen (1954) for angle of approach less than 45° to the normal.

An air stream approaching an obstruction will tend to be deflected along the line of the barrier, (Woelfle 1935, 1936) and Gorshenin (1946). The latter analysed the sheltering effect of tree belts (H $\simeq$ 6 to 10 m.) on winds whose directions were classified into four groups, viz. 0 - 23°; 23 - 45°; 45 - 68°; 68 - 90°, to the line of the belt. The mean percentage reductions in wind speeds within a zone rather more than 20 H wide, were respectively 22%, 30%, 37% and 42%.

Chapter 4

Landscape Features and Air-flow

Section 1 - The frictional influence of landscape as revealed by wind speed relationships

Gross associations between the surface wind $V(s)$ and the geostrophic wind $V(g)$ have been investigated by many workers. Earlier attempts to relate a "free" wind (almost invariably the geostrophic wind), the surface wind and the large scale characteristics of the underlying terrain have been summarised by Shaw (1931, Vol.IV). Many of these results are incorporated into daily synoptic practice and more recently C.E.P. Brooks (quoted by Carruthers, 1943) is known to have suggested the following ratios:-

Type of Terrain	$V(s)/V(g)$
Open sea,	0.60
Low islands,	0.55
Windward coast and neighbouring lowland,	0.50
Leeward coast and neighbouring lowland and sea,	0.40
Open land unsheltered to the West,	0.40
Land sheltered to the west and town sites.	0.30

Marshall (1954) examined surface and geostrophic winds for four stations in the British Isles, values of the ratio $V(s)/V(g)$ ($V(s)$ - an hourly mean value from an autographic record) being analysed according to the direction of the geostrophic wind, time of day and season, temperature lapse rate and pressure distribution. The general average value of $V(s)/V(g)$ varied from 36% for an inland station (Kew Observatory, 52°28'N., 0°19'W.) to 52% for Stornaway (58°11'N., 6°21'W.), and to 67% and 68% respectively for St.Mary's in the Scilly Isles (49°56'N., 6°18'W.) and Bell Rock Lighthouse (56°26'N., 2°24'W.). Diurnal variations were well marked at Kew, and seasonal variations at all four stations, but the considerable variation with geostrophic wind direction was perhaps the most striking common feature. With these and other results in mind, Sawyer (1959), who required some broad relationships, suggested that a value of 0.30 may be used for winds over land surface and 0.60 for those over the sea. In view of the analyses in Part II, it may also be noted that some "anomalies" in the Bell Rock observations with geostrophic wind west of north are considered by Marshall to be a consequence of small lee depressions developing in the Bell Rock area "by the passage of the north-westerly air stream over the mountains of Scotland".

The well known theoretical work of Ekman and G.I. Taylor early this century led to relationships between the wind velocities and the geostrophic wind, and Taylor (1915) was able to show a high degree of agreement between the theory and a set of observations by Dobson (1914). A basic assumption of this early work was that the eddy diffusivity for momentum (Km) is constant with height up to the "geostrophic level" (H), and both Kohler (1933), Rossby (1932) and Rossby and Montgomery (1935) have suggested more realistic assumptions.

Kohler adopts

$$K_m(z) = K_1(z)^m \qquad 0 \leq m < 1 \quad \text{and} \quad K_m(z) = K_1 \quad \text{for } z = 1$$

stability and surface roughness being implicitly allowed for by varying K and m, whilst the second two authors regard the layer as consisting of two sections, the lower one ($0 \leq z \leq h$) in which the logarithmic profile is valid (thus involving z_0 explicitly), and the second $h < z \leq H$; H being the level at which the wind attains its geostrophic value: the frictional drag being set parallel to the wind on $z = h$ and a constant pressure gradient being assumed at all points.

In the 1935 paper, the analysis involved estimates or derived values of z_0 for various sets of data, linked to a general appreciation of upwind conditions, although these authors do not consider it necessary that "the value of z_0 should be chosen so as to be characteristic of the entire trajectory" as they believe "the adjustment of the state of turbulence to the prevailing external parameters is fairly rapid". "Fairly rapid" in this connection must, however, be interpreted consistent with their references to hills 200-300 feet high some miles away. In the course of their work they adopt for a particular set of data a value for z_o of 100 cms. as characteristic of "the eddy producing roughness" of a large European city (mean height of buildings ε = 30 meters, and hence, following Schlichting z_o = 1 meter); and for another series z_o = 3.2 meters for rough hilly country with hills 200-300 feet high leading, in this case, to a value of 0.29 for the ratio of the surface wind at 30 meters to the geostrophic wind. Rossby et al. adopt the formulation

$$\bar{u} = \frac{1}{k} u_* \ln \frac{z+z_o}{z_o}$$

which does not employ a second constant d now favoured for very rough surfaces. Sutton (1953, p. 249) regards Rossby's extension of the rough surface theory to general terrain as "perhaps ....beyond the point of legitimate application".

Possibly the extension of this theory to an assemblage of sharp edged vertical surfaces, such as occurs in a built-up area, may be less open to criticism than its extension to terrain where the contours are much smoother. Rossby and Montgomery also retain the relationship $z_o = \varepsilon/30$ although, as has already been noted (p. 12), investigations over vegetated surfaces imply an appreciably larger fraction, e.g. $\varepsilon/3$ or $\varepsilon/10$

The above investigations all postulate a constant pressure gradient up to the geostrophic level, but some recent observations by Sheppard, Charnock and Francis (1952) indicate that in the lower layers of the oceanic westerlies the assumption does not appear to be valid, although over land surfaces the concept may hold in suitable weather situations;

and it is their view that, in general, physical factors will often combine to render the classical model a useful one.

Sawyer (1959) has recently returned to the problem and published estimates for a coefficient K^* defined by

$$\tau_0 = K^* V_{(g)}^2$$

having assumed (see p. 48) that $V_{(s)} = 0.30\ V_{(g)}$ for land surfaces and $V_{(s)} = 0.60\ V_{(g)}$ over the sea. In Tables 1 and 2 of his paper a five-fold increase in K^* is indicated over surfaces varying from grassland to mountainous terrain. He also concludes from an examination of observations by Seelinger (1938) that "the drag is greater over the rougher topography of Central and Southern Germany than over the northern plain in the ratio of 2 or 3 to 1."

Except for gross contrasts of the type mentioned in the foregoing paragraphs, the search for useful links between upper and surface winds on the one hand and surface geometry on the other seems likely to be difficult. The need for some basic reference velocity which is fairly readily obtainable is, however, so insistent in studies of the type considered in this work that a continual reappraisal of the situation is necessary.

Jensen (1954) has carried out what appears to be a unique investigation to establish the gross effect on surface wind of two contrasting stretches of landscape in Jutland.

Two measurement lines were established running from the west to the east coast of Jutland: Line 1, which was some 76 km. long, ran across a tract of country sparsely provided with hedges and belts. Twelve measurement points were equipped with a recording anemometer with head 2m. from the ground. The extreme points 1 and 12 were situated on islands. The second east-west line with 11 measuring points and rather more than 100 km. in length, lay about 50 km. further north over a relatively hilly tract of country in mid-Jutland well provided with shelter belts, plantations and hedges. Jensen first evolved a "roughness coefficient" which, after some preliminary wind tunnel work, was defined as the sum per unit area of the product of (i) the height of hedgerow etc., (ii) its cross wind (i.e. projected) length, and (iii) a density figure - the latter on a scale 1 to 5 (from "very open" to "very close"). The coefficient for Line 1 was 0.003, and for Line 2, 0.070 (no attempt was made to allow for the hilly nature of the ground in Line 2). Measurements were carried out in December 1949 on two occasions when a uniform pressure field with overcast sky and westerly winds (Force 3-6) prevailed.

Jensen then states that it was possible "to observe a characteristic combination of velocity variations for up to 70 km of the line". The meaning of this and the mode of its identification are obscure, but it is clearly some well-marked feature of the wind pattern (large scale eddy systems, or perhaps frontal passages) which can be traced from one anemometer record to the next. He found that the ratio of the speed of transmission of these features to the geostrophic wind was 0.86 and when reading wind speeds from the chart "the time interval has been shifted corresponding to the time it takes the velocity variation to cover the distance". Mean velocities and the mean value of the maximum velocities were extracted for every quarter of an hour throughout the measuring period. Some results, allowing for the time shift just referred to, are:-

Line 1. mean speeds reduced to 80% of the incident speed within the first 10 km.; to 58% by the penultimate point, but following a further 10 km. fetch over open sea rose again to 75%. Mean maximum speeds not influenced so much (respectively 90%, 60%, 85-90%).

Line 2. mean speed reduced to about 50% of incident value after 20-30 km. fetch over this rougher terrain, remaining at about that level and leaving the east coast at about 50%. Again mean maximum speeds were less affected (respectively 55-60% and 60-75%).

2 From our present standpoint the most interesting observation is that a steady state was apparently reached after 20-30 km. fetch over the rougher transect (Line 2) with a wind speed 50% of the incident value, whereas over the smoother terrain there was a continued decrease to the eastern sea coast - suggesting that the new boundary layer had not been completely established - followed by a marked response to the change from a land to a sea surface.

Below are given ratios to the geostrophic wind speed of the mean speed at 2 metres averaged over the whole track, and as adjusted to the standard height of 10 m.

	Sea	Line 1.	Line 2.
$V(2)/V_g$	.38	.29	.21
$V(10)/V_g$	.51	.39	.28

The relative values are more significant than the absolute ones, but, even if fortuitous, the agreement between the ratio of 0.28 for the hilly terrain and Rossby and Montgomery's figure of 0.29 for $V(30)/V_g$ for similar terrain, is worth noting.

Section 2. Ecological indexes of "exposure".

In general terms it is reasonable to suppose that the natural flora and fauna of any area will respond to the climatic environment and will exhibit certain characteristic distribution patterns and/or growth forms. Such relationships in hilly areas have led ecologists to the concept of the "moorland edge". This boundary is usually identified with the "upper" limit of profitable cultivation and clearly economic and other non-physical factors significantly influence its position. Nevertheless, at any particular time, the relative position of the edge at different places might well depend largely upon physical factors. A striking example of the large scale variations in this boundary is mentioned by May and Willis (1942) in their account of Montgomeryshire prepared for the Land Utilisation Survey. In that county the high ground runs roughly north to south, as a result the eastern part of the county is in a "rain shadow" and relatively sheltered from the prevailing west wind: the "moorland" edge is found under 400 ft. in places to the west of the high ground, but is rarely below 800 ft. on the east side.

Reference has already been made to the work of F.H. Whitehead of Oxford (p. 14). In his 1957 paper he reported further field studies and parallel wind tunnel investigations with a variety of plant species which he was able to classify into three groups, viz.:-

(i) wind "evaders" characterised by a very compact growth form never more than a few cm. high and hence never subject to very strong wind,

(ii) wind tolerant plants,

(iii) wind susceptible plants.

which?

These characteristics could be linked with the moisture content of the leaves, and it was found that photosynthesis, and hence growth, ceased if too great a deficit below saturation was built up in the leaf tissue. Such a deficit with certain species in group (iii) could follow a three-hour exposure to winds of 40 m.p.h. and T $\simeq$ 25^{o}C. After 10 hours of this treatment the chance of subsequent recovery - even given 8 hours of darkness with quiet conditions and relatively low temperature - was remote. On the other hand, recovery could be expected under natural conditions after $6\frac{1}{2}$ hr exposure to 25 m.p.h. wind. Considerations of this sort led him to conclude that in areas such as the Appenines, subjected two or three times a year to very high winds, the species in question could never survive and in practice would never be found. In areas subjected to more frequent but less violent winds, however, the species will tend to develop growth forms, e.g. thicker leaf, more woody tissue and dwarf characteristics, which enable it to recover from transient wind stress. Obviously both the absence, and deformed growth, of this particular species could provide some index to wind exposure and investigations on these lines promise to render more precise, studies such as described in the following paragraphs.

In the last 20 years or so there has been a renewed interest in the possibility of mapping air-flow by studying the geometry of tree distortion. Successful work on these lines has been done in Japan (Sekiguti, 1951) whilst in the British Isles Thomas (1958(a)) has recently published an account of an investigation in which dominant air-flow patterns in the hinterland of Aberystwyth were traced by an examination of the crown deformation of hawthorn. A suggestive feature of the Japanese work is the claim (Misawa K., Misawa, H., 1954) that seasonal wind régimes can be identified by examining characteristic asymmetries of different species which are wind sensitive at different times of the year. This work suggests a distinction between the prevailing wind, and what might be termed the "dominant" wind. The characteristic "flagging" of trees away from the east on the coastal plains of N.E. England seems to support such a distinction, and it would appear that either, when the trees are wind sensitive the dominant wind is from the east, or that only easterly winds are associated with other factors which lead to a deformation of growth: the changes in seasonal liability of farm livestock to wind stress give rise to similar questions. One of the most striking examples of the use of tree distortion as an index of wind exposure is that described by Putnam (1948) in which suitable sites for wind turbines in New England, U.S.A. werepartly selected by reference to ecological criteria. Putnam recounts that it was possible to relate the height of certain tree species (Canada balsam) to a mean wind speed at "specimen height" from which a five stage classification of exposure was developed.

Efforts by the Electrical Research Association to find suitable sites in the British Isles for wind turbines have been described by Golding (1955). They adopted the criterion that any hill area carrying larger vegetation than low heather and rough grass is not sufficiently exposed to require serious consideration, and add that "in Great Britain neither trees nor any vegetation growing to heights greater than a few inches are ever found on hills with annual mean wind speeds above about 23 m.p.h."

In the past few years the Forestry Commission Research Branch have been attempting to map qualitatively the liability of an area to wind exposure by measuring the decrease with time in the area of a small flag of standard size and material. Rough associations of the "tattering" of the flag with gross run of wind have been found at selected sites (unpublished memos. by Lines, R.F., 1958 and Thomas, 1957). The possibility of surveying a large number of areas in respect of exposure with such a simple device encourages the hope that it may give reproducible results.

Following some years' intensive field work, Golding and his associates, while noting "the comparatively little precise information on wind over hills, on the effect of differing contours and the screening effect of high ground in the vicinity", yet consider that "it is possible for an experienced observer ..... to place individual hills in a scattered group in order of windiness with considerable accuracy", and further state that their field work has led them to "place considerable emphasis on the value of experience or 'know-how'".

It is encouraging to read that the trained observer can develop consistent, although qualitative and subjective, criteria for estimating wind exposure, and it is reasonable to expect that this can be replaced step by step by more quantitative and objective knowledge. In this process it is most likely that surface geometry will play a dominant role and that methods of analysing wind observations as outlined in Part II should prove a valuable contribution. The impossibility of setting up anemometers at every site of interest, and even given instruments, the impracticability of waiting long enough for representative data to be obtained, make it essential to exploit information from all sources. At this present stage, for the ecological approach to be fruitful, it is only necessary that ecological and meteorological techniques should lead to a concordant assessment of exposure.

Section 3. Some evidence relating to topographical shelter by ground contour.

It is obvious that ground contour itself may provide shelter from the wind, and as a matter of experience it is confirmed that such shelter can suffice to meet the needs of a wide range of farm livestock against most weather hazards - one exception is heavy snowfall, snow tending to accumulate in the very areas which enjoy topographical shelter.

If an obstacle is to provide shelter, the formation of a wake is necessary and we therefore need information to judge whether or not, in a particular situation, there will be a breakaway of the boundary layer.

Recent theoretical work and observations on air-flow over mountains have been summarised by Corby (1956) who remarks that "separation of flow will be more likely if the lee shape comprises a sharp edge and a precipitous drop ... than over a smoothly rounded hill". Further, any meteorological factor leading to downslope winds, notably radiationally cooling of a lee slope, or the cooling of a mass of air by rainfall, will discourage break-away at the summit. Scorer (1955), on whose theoretical work Corby largely draws, notes in addition "convection makes it (separation) more likely over lee slopes but inhibits it at the top of windward slopes". To supplement these very general points some relevant scattered observations are listed below: generally the topographical model implied is that of an infinite horizontal cross-wind edge:

(i) Woelfle (1950) states that lee slopes of less than 8^{o} (less than one in seven) are unprotected.

(ii) Snow tends to accumulate in an eddying wake (the exceptional instances are those of intense localised scouring eddies). An investigation in the U.S.A. (Anon.1950) revealed that railway cuttings of 11 ft. and 18 ft. deep were "self-cleaning" (viz. of snow) when the slopes of the sides were respectively $<$ 1 in 4 and $<$ 1 in 6.

(iii) Slater (1954) quotes observations by Welzenbach suggesting that airborne material, e.g. snow, insects, will be swept clear from lee slopes less than 18 degrees to the horizontal.

(iv) Manley (1945) suggests that a slope of one in four to one in six plays a significant part in the production of the well-marked "helm" wind phenomenon in Crossfell, Cumberland, although the subsequent work of Scorer tends to shift the emphasis away from a purely mechanical explanation.

(v) Scrase (1930) found no evidence from bi-vane records of gustiness due to a smoothly contoured ridge height H, distance 30 H upwind.

(vi) Gorshenin (1946) gives the relative wind speeds of a wind blowing upslope (wind at base = 100%) and downslope (wind at summit = 100%) of a small hill which rose to 65 metres in 650 metres. Wind speeds were measured at each change of slope, the shortest track distance was about 40 metres on one of the steepest portions and the longest, 100 metres. Observations appear to have been taken over some 20 days in 1941 with electrically controlled Fuess anemometers. The data in Table 4(1) have been derived from some diagrams in the original paper.

Table 4(1). Relative Wind Speeds (per cent.) measured at Various Points on a Hill Slope for Upslope and Downslope Winds

(N.B. $\frac{2-9}{100}$ refers to a point at which a slope of 2/100 changes to one of 9/100)

(From Gorshenin, 1946)

	Base			Degree of Slope				Summit
	$\frac{0-2}{100}$	$\frac{2-9}{100}$	$\frac{9-7}{100}$	$\frac{7-17}{100}$	$\frac{17-11}{100}$	$\frac{11-9}{100}$	$\frac{9-7}{100}$	$\frac{7-5}{100}$
Wind blowing up slope (Wind up slope →)	100	100	112	121	127	127	131	123
Wind blowing down slope (← Wind down slope)	91	89	91	85	81	95	98	100

The greatest speed change of upslope wind occurs as the slope increases from 7/100 to 17/100. Of more importance in the present context is that the wind speed drops rather sharply as the down slope increases from 11/100 to 17/100 recovering somewhat when it levels off to 7/100. This suggests that the boundary layer leaves the lee slope at about 1/10, which is consistent with the other evidence already given. A provisional generalisation would therefore seem to be that slopes of angle less than about $\tan^{-1} 1/9$ are unprotected.

It needs to be stated that the confluence of two smooth downward flowing streams of air from adjacent slopes would obviously give rise to an eddying zone, although it seems unlikely that this will result in any form of "topographic" shelter. As mentioned earlier down draughts can also occur with much steeper slopes (at any rate up to 42 degrees) particularly in hilly and mountainous regions, whose geometry permits differential irradiation of surfaces by the sun.

Slopes of more than one in ten present difficulties to arable farming and thus it seems clear that in the gently rolling countryside where this type of enterprise, and also intensive dairying, are most profitable, ground contour is unlikely to offer any shelter from the wind. It is, of course, otherwise in the most typical stockrearing areas.

Clearly as the length of a finite crosswind barrier is reduced end-effects will more than proportionally reduce the extent of the protected zone, and the zone will be additionally decreased the more the topographical feature assumes a conical form.

The possibility of useful guidance being provided by wind tunnel studies with models has been exploited from time to time, and in this connection it is understood (Hogg, 1957) that results sufficiently accurate for land planning purposes are being obtained in Germany by a combination of laboratory study of flow patterns over models of the countryside, with parallel field measurements.

P A R T II

THE ANALYSIS OF SURFACE WIND DISTRIBUTIONS

Chapter 5.

Some Statistical Techniques for the Analysis of two-dimensional Wind-fields.

Section 1. Introduction.

The working hypothesis guiding the present investigation is that the various agencies influencing surface air-flow will give rise to certain characteristic patterns in the wind-rose. Provisionally it is assumed that two sources of variability may be recognised, viz. synoptic and mechanical, and that the mechanical constraints, i.e. the topographical features, will cause certain distortions in a pattern basically determined by the larger scale synoptic factors.

Clearly a mechanical constraint may not operate unless a certain synoptic situation prevails, and a given topographical feature may well result in different types of distortion according to the prevailing weather; whilst amongst a number of cases in which the two types of physical control interact strongly are the lee depressions which form in mountainous areas, and the large scale mountain-valley wind systems. All that is expected from the present study is the emergence of certain qualitative criteria which may aid in the classification of surface wind observations.

Many peculiarities of wind distributions have, of course, been "explained" by reference to upwind topography - some earlier investigations have been summarised by Shaw (1931), and many others may be found in published and unpublished studies of "airfield meteorology". However, the extensive application of statistical techniques to wind analyses since about 1940, and the demands for more information from many types of enterprise, combine to render a reassessment of the situation very desirable.

The first step is obviously a detailed examination of some techniques for the analysis of the wind field, and of their application to a few special cases; this task is undertaken in the present thesis.

Amongst the techniques which are considered in the analyses to be described, some are well known, others are mentioned in the literature, but not in detail, whilst others have been developed for this particular investigation.

In the "Handbook of Statistical Methods in Meteorology" by C.E.P. Brooks and N. Carruthers (1953), an account is given of the methods of analysing the scalar distribution of wind speed by means of Pearson's curves, the Poisson and modified Poisson Series (pp. 80-82 and 315), the log-normal and the adjusted normal (p. 112 et seq.), and reference is also made to a paper by Dinkelacker (1948) in which the reduced variate V/V(m) is used, V(m) being the median speed. In addition Kojima (1957) has shown that the scalar distribution of wind speed for a number of stations in Japan is well represented by a gamma-distribution in which the important parameter is $s/\overline{V(s)}$, s being the standard deviation of the scalar speeds and $\overline{V(s)}$ the scalar mean.

The above methods were applied to two cases, but the fit with the appropriate theoretical curves did not seem sufficiently good to justify pursuing these ideas intensively at this stage, in view of the possibility of a fruitful direct attack on the two-dimensional wind field. Consideration was therefore given to the application or adaptation of the two parameter specification of wind distributions now successfully applied to homogeneous sets of observations in the upper atmosphere. This technique rests upon the assumption that the vector ends form a bivariate normal distribution about the end of the vector mean $\overline{\mathbb{V}}$, and that the distribution is symmetrical (or "circular") and hence specified by one additional parameter $\sigma_{(0)}$, the standard vector deviation. The method is described in detail in the "Handbook" mentioned above, and is further elaborated, with an example worked out in full (using numerical interpolation where necessary), in "Upper Winds over the World" - Geophysical Memoir No. 85 (Brooks, C.E.P., Durst, C.S., Carruthers, N., Dewar, D. and Sawyer, J.S., 1950): (this work will be referred to repeatedly and for convenience will be designated "GM.85" in the following pages). Doubt was early thrown on the practice of regarding a set of wind observations as inhomogeneous if the distribution was not circular, and Scott (1956), amongst others, has produced evidence of genuine and significant ellipticity in certain bi-variate distributions; in such cases a third parameter is required to specify the field.

It is revealed in the present study that the derivation of, and certain relationships between, parameters are not as closely dependent on
222. the bi-variate Gaussian assumption as implied in the "Handbook" and "GM.85", and this finding is held to justify an attempt to use the two parameter model for surface wind distributions which clearly diverge markedly from a normal circular distribution (designated "NCD" in this work).

Any wind distribution will furnish values of the parameters $\overline{V(s)}$ $|\bar{\mathbf{V}}|$ $q\left(\equiv |\bar{\mathbf{V}}| / \overline{V(s)}\right)$ $\sigma_{(0)}$, etc. and it is a matter for decision as to which are to be taken to specify the corresponding NCD. On the grounds that the value of q is more likely to be generally available than that of $\sigma_0 / |\bar{\mathbf{V}}|$ (for the NCD there is a one-one relationship between these two parameters), q was usually regarded as the independent variable, and the theoretical model developed using the value of $\sigma_0 / |\bar{\mathbf{V}}|$ appropriate to the NCD of stated q (the notation $\left[\sigma_0 / |\bar{\mathbf{V}}|\right]$ was adopted to denote this particular value of the parameter).

If $|\bar{\mathbf{V}}|$ be regarded as fixed for a particular distribution then $[\sigma_0] \neq \sigma_0$ and it is clear that other procedures will involve other inconsistencies. There is need for further work on these topics, but for the first approach to the analysis of the two dimensional field it appeared adequate to accept either q, or $\sigma_{(\theta)}/|\bar{V}|$ as the independent variable, or even to use the observed values of both q and $\sigma_{(\theta)}/|\bar{V}|$ to construct certain components of the theoretical model. Frequency density diagrams constructed from the actual observations were compared with those for the equivalent NCD. An extension of the model was then considered in which displacement D, or flow, due to winds lying within specified limits of speed and direction replaced the frequency of winds - the suggestion being that distortion in the "D" field due to topography might be expected to be physically more significant than the associated anomaly of the frequency density pattern.

It was judged expedient to pay particular attention to the displacement and frequencies within certain directional ranges, for the reason that, whereas the continuous recording of wind velocity is only likely to be possible at a few sites, a denser network in which relatively simple instruments (some with purely mechanical linkages) can be employed to give "run of wind" in each of eight or so directions and/or total frequencies from such directions may be a practical possibility.

These methods were used to analyse surface wind data from four well exposed stations, it being supposed that such an analysis might reveal the way in which synoptic factors manifest themselves in the two dimensional wind field, in the absence of marked topographical or other mechanical constraints. The choice of station was further limited by the obvious need to rely only upon high quality observations covering the twenty-four hours of the day.

The observational series eventually selected were for :-

(i) Ocean Weather Ship "Juliet" (OWS "J") mean position $52\frac{1}{2}^{\circ}$H; 20°N, vane between 50 ft. and 65 ft. above sea level. 1950-52: 1953-55.

(ii) "Northice" Greenland; $78^{\circ}04'$N, $38^{\circ}29'$W. height 2343 m. above m.s.l., vane 10 m. above surface. 1st November 1952 to 15th July 1954.

(iii) Bell Rock Lighthouse, $56^{\circ}26'$N; $2^{\circ}24'$W. vane 130 ft. above m.s.l.1951-55.

(iv) Lerwick Observatory, $60^{\circ}8'$H; $1^{\circ}11'$W; vane 310 ft. above m.s.l.1951-55.

Details are given later (Chapter 8) and it suffices at this stage to note that the stations are representative respectively of:-

(i) a free exposure, presumed independent of topographical influences;

(ii) a land station on a level, completely featureless, but slightly sloping, plane;

(iii) a particularly well exposed station by ordinary Meteorological Office convention;

(iv) a well exposed "standard" station.

Section 2. The analysis of two-dimensional wind distributions - The normal circular distribution

(i) A mechanical analogy

On considering methods currently adopted for the analysis of the wind field it was apparent that operations are involved identical with some employed in the study of centroids and coefficients of inertia of a plane distribution of point masses. The formal analogy - recognised earlier by Van der Stok (1907) - seems a helpful one, and it was thought that by bearing it in mind a number of features, possibly obscured in the now customary approach, might be made more evident.

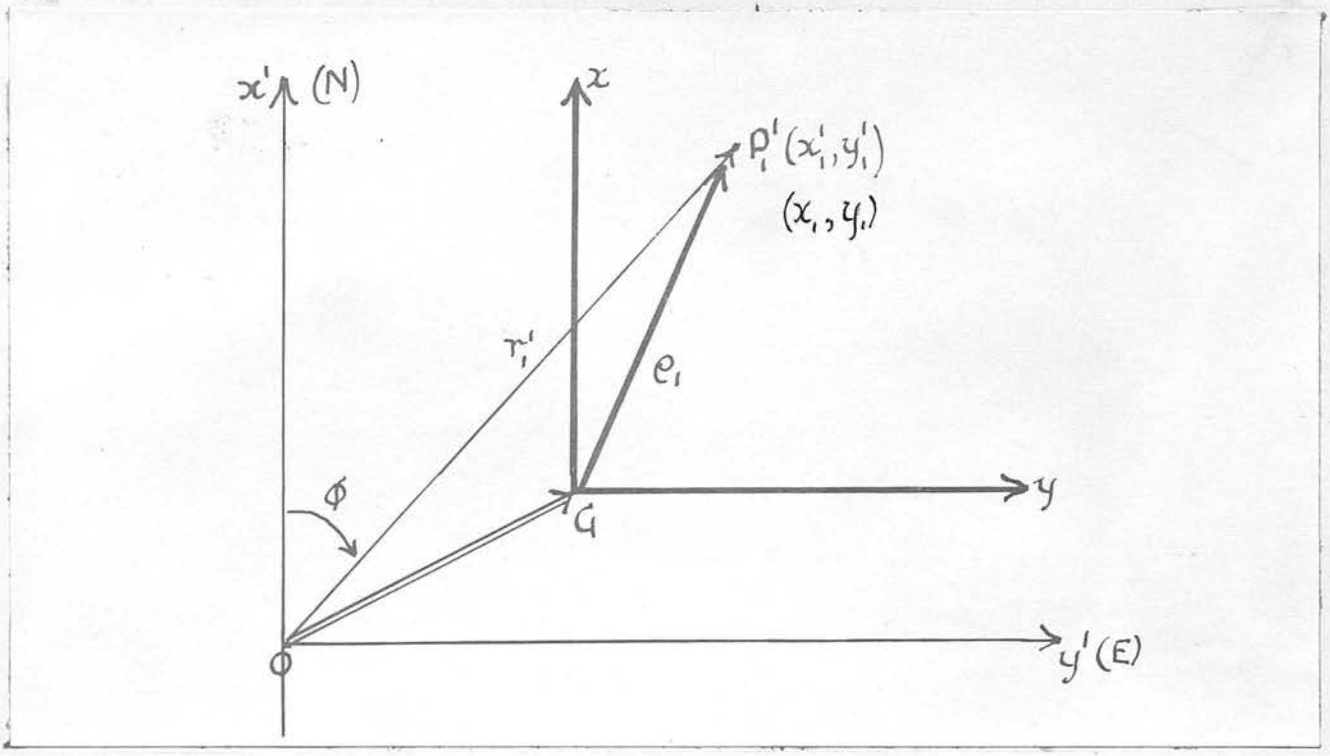


Figure 5(1). Coordinate system for the analysis of wind distributions

In Fig.5(1) let O be the origin, Ox' towards north, Oy' towards east, and suppose ϕ be measured clockwise from Ox', so preserving the usual climatological convention.

Let the N wind vectors OP_1', OP_2', OP_3', OP_N' be replaced by unit point masses at $P_1'(x_1' y_1')$ etc, then clearly the end of the vector mean is at G - the mass centre (in the analysis of wind distributions, this point will later be designated **A**, as is customary).

If $OP_1' = r_1'$ etc. then from the theorem of parallel axes

$$\sum_1^N r_1'^2 = \sum_1^N \rho_1^2 + M.\,OG^2$$

where $\rho_1 = P_1'G$, $\rho_2 = P_2'G$, etc. and $M = \Sigma m \equiv N$

and hence defining $\sigma_{(0)}^2$ by

$$\Sigma \rho_i^2 = N\sigma_{(0)}^2$$

we have

$$\sigma_{(0)}^2 = \frac{\Sigma r_1'^2}{N} - OG^2 \equiv \frac{\Sigma |OP_1'|^2}{N} - |OG|^2$$

where $\sigma_{(0)}$ is now seen to be the standard vector deviation.

If now the assemblage of point masses is referred to G as origin and axes Gx and Gy, and defining as customary

$$A = \Sigma m_1 y_1^2 \qquad \equiv \text{MI about Gx}$$

$$B = \Sigma m_1 x_1^2 \qquad \equiv \text{MI about Gy}$$

$$F = \Sigma m_1 x_1 y_1 \qquad \equiv \text{PI about Gx Gy}$$

then the coefficients of inertia A_o, B_o, F_o about axes making angles α and $\alpha + \pi/2$ with Gx are

$$A_o = A\cos^2\alpha + B\sin^2\alpha - 2F\sin\alpha\cos\alpha$$

$$B_o = B\cos^2\alpha + A\sin^2\alpha + 2F\sin\alpha\cos\alpha \qquad \ldots\ldots\ldots 5(1)$$

$$F_o = \frac{1}{2}\sin 2\alpha\,(A-B) + F\cos 2\alpha$$

Consideration of the expression

$$(A_o =) \quad A\cos^2\alpha + B\sin^2\alpha - 2F\sin\alpha\cos\alpha \equiv M\,\varepsilon^4/\rho^2$$

leads to the momental ellipse which, since it is now referred to G as origin, is the central ellipse. The angle α is arbitrary and may be selected so that

$$F_o = 0$$

and the new axes then become the "principal axis of inertia" of the plane distribution, centre G.

A two-dimensional mass distribution of any shape, or an assemblage of point masses lying within any irregular boundary, has an associated central ellipse. There is no necessary similarity between the geometry of the mass distribution and the shape and orientation of the central ellipse, although if a plane distribution of point masses has approximate symmetry about the mass centre and is confined within a more or less elliptical boundary, the major and minor axes of this "material" ellipse and of the central ellipse will be roughly parallel and proportional.

In the familiar methods of analysing wind observations, a set of relationships similar to those in Eq. 5(1) ~~are~~ derived, viz. is

2 $$\sigma_{(Y)}^2 = \sigma_{(y)}^2\cos^2\alpha + \sigma_{(x)}^2\sin^2\alpha - 2R\,\sigma_{(x)}\sigma_{(y)}\sin\alpha\cos\alpha \;\;\ldots\ldots\ldots 5(2)$$

where $\sigma_{(x)}$ and $\sigma_{(y)}$ are standard deviations, and R the correlation coefficient between deviations, of wind components along Gx and Gy. A customary next stage may well involve comparing the density of actual vector ends bounded by ellipses concentric to that derived from Eq. 5(2),

2 with those assuming the bivariate normal assumption, but published accounts which?

do not sufficiently emphasise that the derivation of an ellipse defined by certain statistics of the wind field in no way depends upon that field having a particular density distribution. This point is relevant when considering Mauchly's (1940) method for testing whether or not a wind distribution is elliptical.

(ii) <u>Mauchly's test for "ellipticity"</u>

This test is recommended by Brooks and Carruthers (1953) to establish whether an apparently elliptical distribution is significantly divergent from a circular one.

If $\sigma_{(x)}$, $\sigma_{(y)}$ and R have the meanings already assigned to them, Mauchly (1940) states that, given a normal circular distribution, the probability of obtaining a value L_ε as low as that actually obtained is

$$(L_\varepsilon)^{N'-2}$$

where

$$L_\varepsilon = \frac{2\,\sigma(x)\,\sigma(y)}{\left(\sigma(x)^2 + \sigma(y)^2\right)} \sqrt{1-R^2}$$

and N' is the number of <u>independent</u> observations.

Two useful aids for computation may be noted at this stage. First, Mauchly in his 1940 paper notes that the standard deviations along the principal axes ($\sigma(a)$ and $\sigma(b)$ (say)) are the roots $\sqrt{k_1}$, $\sqrt{k_2}$ of the equation,

$$\begin{vmatrix} \sigma(x)^2 - k & \sigma(x)\sigma(y)R \\ \sigma(x)\sigma(y)R & \sigma(y)^2 - k \end{vmatrix} = 0$$

In the second place we have the relationships:

$$\sigma(a)^2 + \sigma(b)^2 = \sigma(x)^2 + \sigma(y)^2$$

$$\text{and} \quad \sigma(x)\sigma(y)\sqrt{1-R^2} = \sigma(a)\,\sigma(b)$$

and hence

$$L_\varepsilon = \frac{2\,\sigma(a)\,\sigma(b)}{\sigma(a)^2 + \sigma(b)^2}$$

$$\text{and} \quad R^2 = 1 - \frac{\sigma(a)}{\sigma(x)}\,\frac{\sigma(b)}{\sigma(y)}$$

It is desirable to note that, although the value of depends upon parameters equivalent to those of the central momental ellipse, the significance test is stated to presuppose the parent population to be a bivariate normal one.

The significance of L_ε depends critically upon N', the number of independent observations, and several authors provide some information on the ratio of N to the actual number of observations of wind velocity made.

(i) Brooks and Carruthers (1953, p.194) state that: "Winds recorded once or more times a day in runs of consecutive days may be assumed equivalent to D/2 independent observations where D = number of days". This appears to be based upon a study of the time distribution of gales from which it is probable that a denominator greater than 2 is appropriate for observations of mean hourly wind speeds.

(ii) Court, A. et al. (1957) suggest that N observations are equivalent to N/4 independent observations. These authors are concerned with upper winds and it may be assumed that they are dealing with observations of frequency once or at most twice daily.

(iii) Durst, C.S. (1954) suggests the law

$$r = \exp(-at)$$

holds for the stretch-vector correlation coefficient where $a = 6.9 \times 10^{-6}$ and t is in seconds. If we assume $r \leqslant 0.1$ is negligible, then r is reduced to less than this value in four days.

(iv) Wind is intimately linked with the pressure field, and here again the serial correlation is reduced to a negligibly small quantity after about five days.

(v) Scott, J. (1957) found at Singapore for winds above 10,000 ft. that "an interval of two days is probably more than sufficient for the independence of S-N components.... but for E-W components the interval had to be increased to one week before an acceptably small serial correlation coefficient was obtained."

The above evidence suggests that only observations at about four day intervals are likely to be independent and on this basis various observational series analysed later give rise to between 20 and 50 "independent" observations. In Table 5(1) are set out values of L_ε for various ratios of $\sigma_{(a)}/\sigma_{(b)}$, $\sigma_{(a)} < \sigma_{(b)}$ and degrees of freedom (N'-2) at the 1% and 5% significance levels.

<u>Table 5(1)</u> <u>Values of L_ε and Associated Upper Limits for $\sigma_{(a)}/\sigma_{(b)}$; $\sigma_{(a)} < \sigma_{(b)}$ for Selected Number of Degrees of Freedom for Significance at the 1% and 5% levels</u>

Significant level	Parameter	Degrees of Freedom (N'-2)					
		21	28	32	38	45	100
1%	L_ε	.803	.848	.866	.886	.903	.955
	$\sigma_{(a)}/\sigma_{(b)}$	.50	.55	.58	.61	.63	.74
5%	L_ε	.867	.899	.911	.924	.936	.970
	$\sigma_{(a)}/\sigma_{(b)}$	.58	.63	.64	.67	.69	.78

For the range $20 < N'' < 100$ where $N'' = N' - 2$ the relationships of N'' with $\sigma_{(a)}/\sigma_{(b)}$ ($\sigma_{(a)} < \sigma_{(b)}$) are closely given by

$$\sigma_{(a)}/\sigma_{(b)} = 0.345 \log N'' + 0.055$$

for the 1% level

5(3)

$$\sigma_{(a)}/\sigma_{(b)} = 0.283 \log N'' + 0.217$$

for the 5% level

It is relevant to mention also that Mauchly discusses a method of combining evidence from two clouds of points which cannot directly be pooled. A product

$$L \equiv L_{\varepsilon(N)} \times L_{\varepsilon(M)}$$

is formed, and if the number of (independent) observations in both series is the same, then the probability P(L) of obtaining a value of L as low as that actually obtained is given by

$$P\left(L_{\varepsilon(N)} . L_{\varepsilon(M)}\right) = P(L) = L^{N'-2}\left[1 - (N'-2) \ln L\right]$$

The application of this further step, certainly to wind observations, does not seem straightforward. Although the population in two particular cases may both be suspected of being elliptical, yet, unless the characteristic ellipses are similarly orientated in both cases, it may not be at all justified to assume that the ellipticity springs from some physical control such as latitude or topography, and a "significant" degree of ellipticity obtained by considering the two sets of observations together, may be illusory. It is hoped to investigate this problem on a future occasion.

Section 3. Some empirical relationships of the normal circular distribution

(a) The $q : \sigma/|\mathbf{V}|$ relationship

Assuming that homogeneous sets of wind data give rise to a bivariate Gaussian distribution symmetrical with respect to a maximum frequency at the vector mean, i.e. the normal circular distribution of Brooks et al., then for the construction of wind roses for any point in the free air only $\overline{\mathbf{V}}$ and σ are required. (N.B. When no ambiguity arises, the standard vector deviation will be denoted for convenience by σ, dropping the zero suffix necessary earlier in this chapter). Data adequate to do this are often not available in sufficient detail, although $\overline{\mathbf{V}}$ and the scalar mean $\overline{V(s)}$ may be known, and to deal with this situation Brooks and co-workers have established the relationship between q (which they designate the "constancy" and define by $q \equiv |\bar{V}|/\bar{V}_{(s)}$) and $\sigma/|\bar{V}|$. This one-one relationship between q and $\sigma/|\bar{V}|$ for various q was obtained by empirical and graphical methods, although Knighting (1954) later pointed out that two of the important quantities, for whose evaluation Brooks et al. had employed the $q : \sigma/|\bar{V}|$ relationship, viz. the proportionate frequency of winds from 0 – V irrespective of direction, and that of winds lying between ψ_1 and ψ_2 irrespective of speed, may be obtained by employment of some well-known functions. Since, however, the construction of wind roses requires the frequencies of winds lying within any speed limits V_1 and V_2 and any chosen directions ψ_1 and ψ_2, the functions mentioned by Knighting do not completely solve the problem, although they are invaluable for checking the accuracy of the original tables published in "GM 85" and of interpolations, (a more direct method of reconstructing a wind rose for any stated NCD is outlined on p. 99).

Recently the writer came across a tabulation of one of the expressions derived by Knighting (that for wind speed irrespective of direction), and a short note on the application to wind data prepared for publication (Gloyne, 1959), a version of this, appears as Appendix II(a).

In the construction of wind roses σ is required, and if it cannot be computed from the original data it has to be inferred from the $q: \sigma/|\bar{V}|$ relationship, although "the derivation of σ from a value of the "constancy" (i.e. from q) is only strictly accurate if the distribution is normal" ("GM 85", p.14) yet "in view of the probable error in the estimate of q or it seems sufficient to calculate q to the nearest 5%...." ("GM 85", p.4). In the "Handbook" (p.198) mean values of $\sigma/|\bar{V}|$ for particular values of q are given, and in the "GM 85" (p.4) suitable tolerance limits are suggested.

In view of the considerable importance attached to this relationship in this present investigation, and the empirical nature of the derivation employed by Brooks et al., it was thought desirable to undertake a rather detailed examination of the question.

The mean values and reasonable tolerance limits for $\sigma/|\bar{V}|$ corresponding to integral values of q have been given by Brooks et al. and are incorporated in Figs. 5(2) and 5(3) on pages 77 and 78.

According to Knighting (1954)

$$\overline{V(s)} = \sqrt{\pi}\cdot\frac{\sigma}{2}\cdot {}_1F_1\left(-\frac{1}{2}, 1, -\frac{|\bar{V}|^2}{\sigma^2}\right)$$

$$= \sqrt{\pi}\,\frac{\sigma}{2}\left[1 + \frac{1}{2}\left(\frac{|\bar{V}|}{\sigma}\right)^2 - \frac{1}{16}\left(\frac{|\bar{V}|}{\sigma}\right)^4 + \frac{3}{8.6^2}\cdot\left(\frac{|\bar{V}|}{\sigma}\right)^6 - \frac{15}{16.24^2}\left(\frac{\bar{V}}{\sigma}\right)^8 \cdots\right]$$

which converges reasonably rapidly providing $|\bar{V}|/\sigma < 1$.

When $|\bar{V}| \to 0$ (or $|\bar{V}|/\sigma$ is small enough for its square and higher powers to be neglected)

$$\overline{V(s)} = \frac{\sigma}{2}\sqrt{\pi} \qquad \text{or} \qquad \sigma = 1.13\,\overline{V(s)}$$

Since $q = |\bar{V}|/\overline{V(s)}$, we may write

$$\left(\frac{1}{q} \equiv\right) \frac{\overline{V(s)}}{|\bar{V}|} = \frac{\sqrt{\pi}}{2}\,\frac{\sigma}{|\bar{V}|}\left[1 + \frac{1}{2}\left(\frac{|\bar{V}|}{\sigma}\right)^2 - \frac{1}{16}\left(\frac{|\bar{V}|}{\sigma}\right)^4 + \text{etc}\right]$$

$$= \frac{\sqrt{\pi}}{2}\left[\frac{\sigma}{|\bar{V}|} + \frac{1}{2}\frac{|\bar{V}|}{\sigma} - \frac{1}{16}\left(\frac{|\bar{V}|}{\sigma}\right)^3 + \quad\right] \qquad 5(4)$$

giving for large $\sigma/|\bar{V}|$ (say 50 and 100)

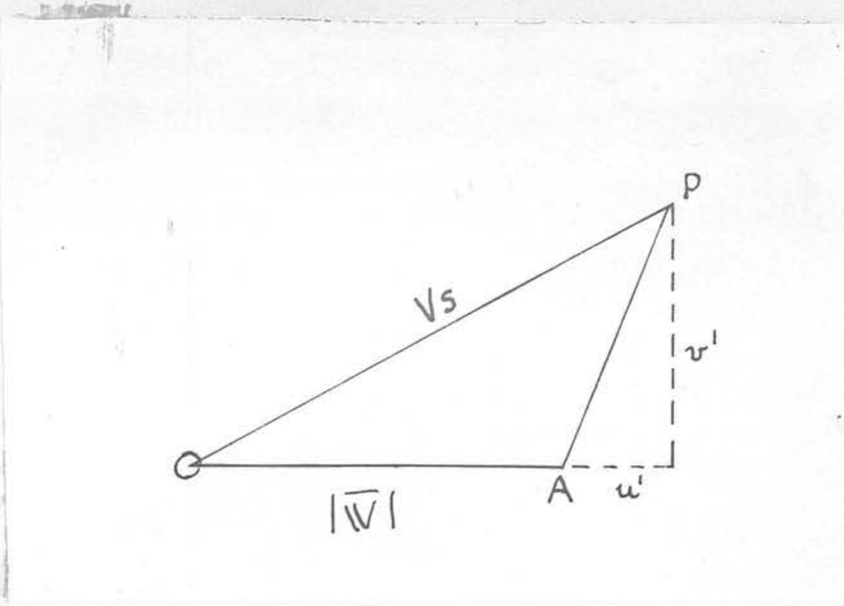
$$\frac{1}{q} = \frac{\sqrt{\pi}}{2}\left[50 + \frac{1}{100} \cdot\right] \simeq \frac{\sqrt{\pi}}{2}(50.01) \quad \text{hence} \quad q = 0.0225$$

and

$$\frac{1}{q} = \frac{\sqrt{\pi}}{2}\left[100 + \frac{1}{200}\cdot\right] \simeq \frac{\sqrt{\pi}}{2}(100.) \quad \text{hence} \quad q = 0.0113$$

compared with .021 and .012 as estimated from the graphs. (It will be noted in Fig. 5(2) that for the range $35\% < q < 80\%$ there is a close linear relationship between $\log \sigma/|\bar{V}|$ and q).

For small $\sigma/|\bar{V}|$ an approximate connection may be obtained as follows:-



Let $V(s)$ be any particular wind such that u' v' are the deviations from the vector mean $\overline{OA}$, then

$$V(s) = \left[(|\overline{V}| + u')^2 + v'^2 \right]^{\frac{1}{2}}$$

$$= |\overline{V}| \left[1 + \frac{2u'}{|\overline{V}|} + \frac{u'^2}{|\overline{V}|^2} + \frac{v'^2}{|\overline{V}|^2} \right]^{\frac{1}{2}}$$

$$\simeq |\overline{V}| \left[1 + \frac{u'}{|\overline{V}|} + \frac{1}{2} \cdot \frac{v'^2}{|\overline{V}|^2} \right]$$

ignoring those powers of the deviations higher than the second.

Hence

$$\overline{V_s} \simeq |\overline{V}| \left[1 + \frac{1}{2} \frac{\Sigma v'^2/n}{|\overline{V}|^2} \right]$$

Assuming that P lies within a circle centre A, and introducing σ, we have

$$\frac{1}{q} \simeq 1 + \frac{1}{2} \frac{\sigma^2/2}{|\overline{V}|^2} \simeq 1 + \frac{1}{4} \cdot \left(\frac{\sigma}{|\overline{V}|} \right)^2 \quad \ldots\ldots\ldots\ldots\ldots 5(5)$$

If $\sigma/|\overline{V}| = 0.25$ $\quad \frac{1}{q} = 1 + \frac{1}{64}$; hence $q \simeq 98.46\%$

if $\sigma/|\overline{V}| = 0.21$ $\quad \frac{1}{q} \simeq 1 + \frac{441}{4 \times 10^4}$; hence $q \simeq 98.91\%$

if $\sigma/|\overline{V}| = 0.10$ $\quad \frac{1}{q} \simeq 1 + \frac{1}{4} \cdot \frac{1}{10^2}$; hence $q = 99.75\%$

compared respectively with 98.5%, 99.0% and > 99.0% from Fig.5(2). Although equation 5(5) cannot strictly be used for a value of $\sigma/|\bar{V}|$ as large as one half, yet thederived value (q = 93.75%) is negligibly different from that given in the Fig.5(2).

From the above analysis it is clear that as far as the mean curve is concerned, the values empirically derived by Brooks et al. (1953) are confirmed to a very high degree of accuracy by checks based upon Eqs. 5(4) and 5(5).

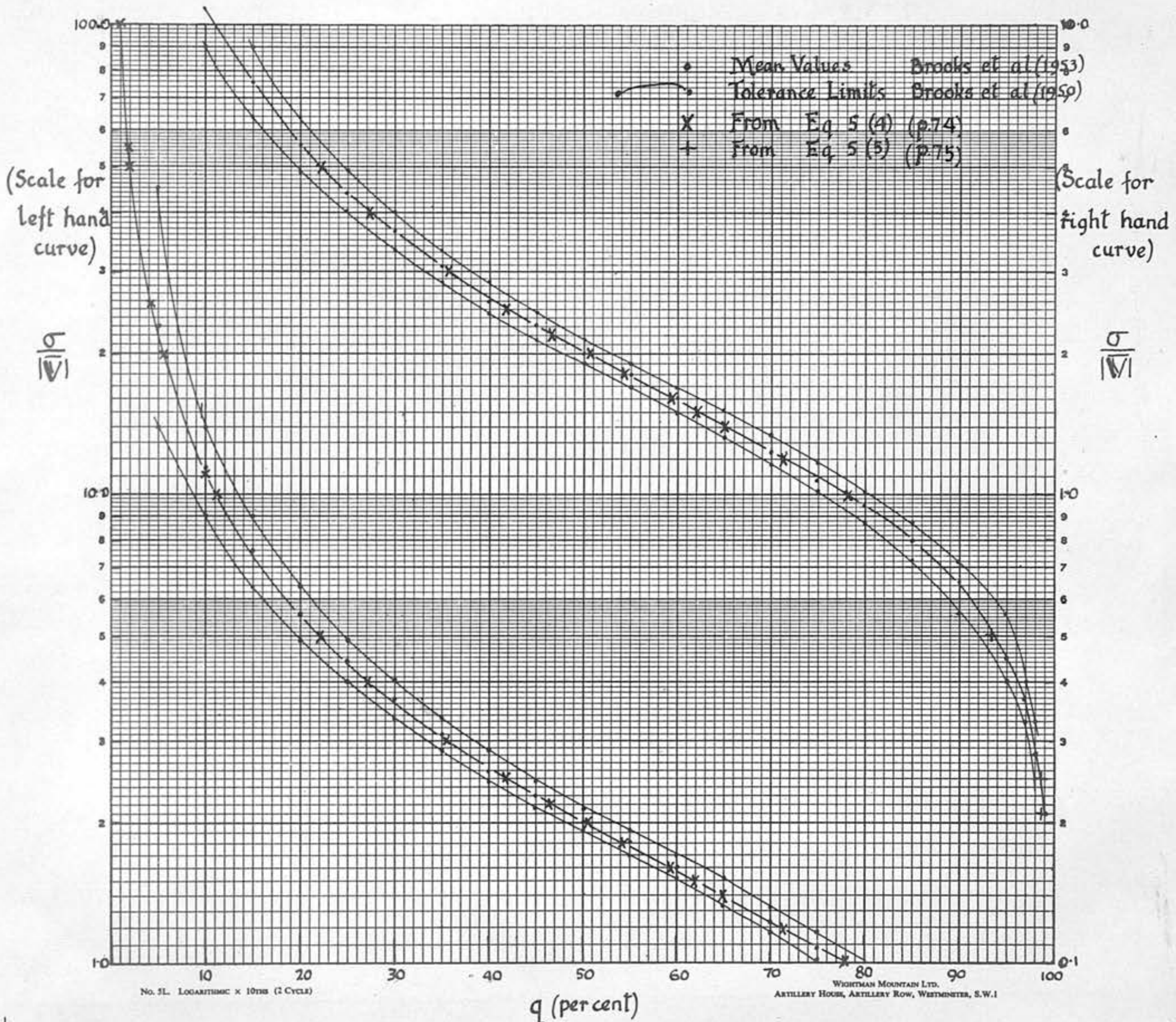


Figure 5(2). To show mean values and tolerance limits for the $q : \sigma/|\overline{V}|$ relationship (after Brooks et al. 1950); together with independently computed values (the mean curve is based upon these computed values).

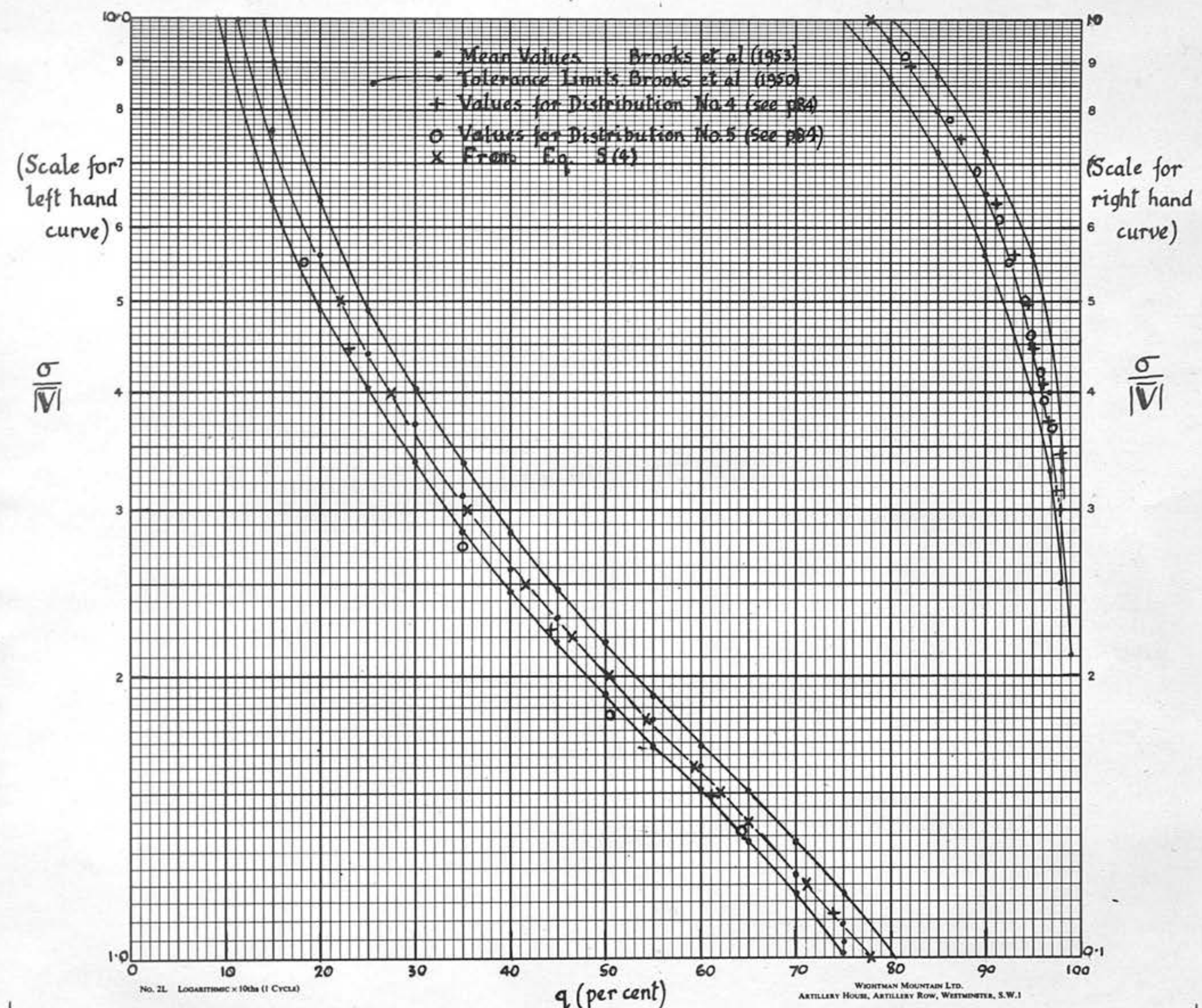


Figure 5(3). To show mean values and tolerance limits for the $q : \sigma/|\bar{V}|$ relationship as in Fig. 5(2); together with computed values for certain hypothetical distributions. (See Chapter 6, Table 6(1), p. 84).

(b) Mean speeds in any direction

It is shown (GM 85) that for a normal circular distribution the mean speed $\overline{V_{(s)(\psi)}}$ in a direction making ψ with the vector mean is given to a close degree of approximation by the expression

$$\overline{V_{(s)(\psi)}} = \tfrac{1}{2}\left[\pi\sigma^2 + |\bar{V}|^2\cos^2\psi\right]^{\frac{1}{2}} + \tfrac{1}{2}|\bar{V}|\cos\psi$$

from which it follows readily that if the average speeds for various ψ are represented by points at distances $\overline{V_{(s)(\psi)}}$ from the origin O in the appropriate directions, then these points will lie on a circle. The centre of the circle is distant $\frac{1}{2}|\bar{V}|$ along the vector mean from O and is of radius R where

$$R = \tfrac{1}{2}\left(\pi\sigma^2 + |\bar{V}|^2\right)$$

In later work the circle is termed the "characteristic" circle.

For certain computations it was convenient to write these expressions in the following alternative forms

$$\overline{V_{(s)(\psi)}} = \tfrac{1}{2}|\bar{V}|\left[\left(\pi\frac{\sigma^2}{|\bar{V}|^2} + \cos^2\psi\right)^{\frac{1}{2}} + \cos\psi\right]$$

from which we note the ratio of the greatest and least speeds will be

$$\frac{\left[\pi\sigma^2/|\bar{V}|^2 + 1\right]^{\frac{1}{2}} + 1}{\left[\pi\sigma^2/|\bar{V}|^2 + 1\right]^{\frac{1}{2}} - 1}$$

Also

$$R = \tfrac{1}{2}|\bar{V}|\left[\pi\sigma^2/|\bar{V}|^2 + 1\right]^{\frac{1}{2}}$$

$$= \tfrac{1}{2}\sigma\sqrt{\pi}\left[1 + \frac{1}{\pi}\cdot\frac{|\bar{V}|^2}{\sigma^2}\right]^{\frac{1}{2}}$$

$$= \tfrac{1}{2}\sigma\sqrt{\pi}\left[1 + \frac{1}{2}\cdot\frac{1}{\pi}\cdot\frac{|\bar{V}|^2}{\sigma^2}\right] \text{ if } \sigma/|\bar{V}| \text{ is sufficiently large}$$

and the mean speed in a direction perpendicular to the vector mean is

$$V_{(s)(\frac{\pi}{2})} = \tfrac{1}{2}\sigma\sqrt{\pi}$$

This is one of the cases noted earlier (p. 63) in which the value of a parameter of the theoretical model (in this instance R) depends upon decisions as to the appropriate independent variable.

(c) A relationship between s (the standard scalar deviation), σ, $|\bar{V}|$, $\bar{V}_{(s)}$

Consider the scalar distribution of speeds $V_{(s)}$, then s^2 is given by

$$s^2 = \frac{\sum |V|^2}{N} - \bar{V}_s^2$$

$$= \sigma^2 + |\bar{V}|^2 - \bar{V}_s^2$$

$$\text{hence } s^2/\bar{V}_{(s)}^2 = \frac{|\bar{V}|^2}{\bar{V}_{(s)}^2}\left[\frac{\sigma^2}{|\bar{V}|^2} + 1\right] - 1$$

$$= q^2\left[\frac{\sigma^2}{|\bar{V}|^2} + 1\right] - 1 \qquad \text{........5(6)}$$

(q being expressed as a proper fraction)

For the limiting case when $|\bar{V}| = 0$, $q = 0$

$$s^2/\bar{V}_{(s)}^2 = \frac{\sigma^2}{\bar{V}_s^2} - 1$$

Thus knowing q and $\sigma/|\bar{V}|$ for any distribution we have immediately the ratio of two basic parameters of the scalar distribution.

The $q : \sigma/|\bar{V}|$ relationships for distributions departing markedly from the normal circular form, differ little from the mean relationships appropriate to the NCD. In contrast Eq. 5(6) is sensitive to such departures.

It is of interest to note, at this point, that for the NCD we have $s/\bar{V}_{(s)} \simeq 0.52$ for the range $0 < q < 50\%$. This result is of interest in connection with Kojima's analysis already mentioned (p. 62) in which he derived the following empirical formula

$$s = 0.5\,\bar{V}_{(s)} + 0.6 \qquad \text{(unit meter per second)}$$

$$\simeq 0.5\,\bar{V}_{(s)}$$

In practice we find that for most surface wind distributions $q < 60\%$ and hence (see Fig.6 (2)) it is a consequence of the normal circular distribution that the scalar standard deviation shall be approximately one half the mean scalar wind speed. As q rises to 80% or so the ratio decreases steadily to about 0.46.

Chapter 6.

Statistical Relationships for certain Hypothetical two-dimensional Distributions.

Section 1. The $q : \sigma / |\bar{V}|$ relationship.

When analysing the wind data presented later in this thesis, it was early noted that, although many of the distributions departed very obviously from the theoretical normal circular form, the $q : \sigma/|\bar{V}|$ relationship was closely obeyed. This prompted a study to see how far the relationship held for certain hypothetical distributions.

It has been shown in Chapter 5 that the standard vector deviation σ is formally identical with the radius of gyration (k) of a plane system of unit masses situated at the vector ends. If N be the total number of wind observations, then, writing M ($\equiv \sum m$) for N, we have from the theorem of parallel axes

$$Mk_1^2 = Mk^2 + Md^2$$

where Mk_1^2 is the moment of inertia about an axis perpendicular to the plane and distant d from the centre of mass (centroid) of the mass distribution (i.e. from the mean centre of the field of vector ends); d is clearly equivalent to $|\bar{V}|$

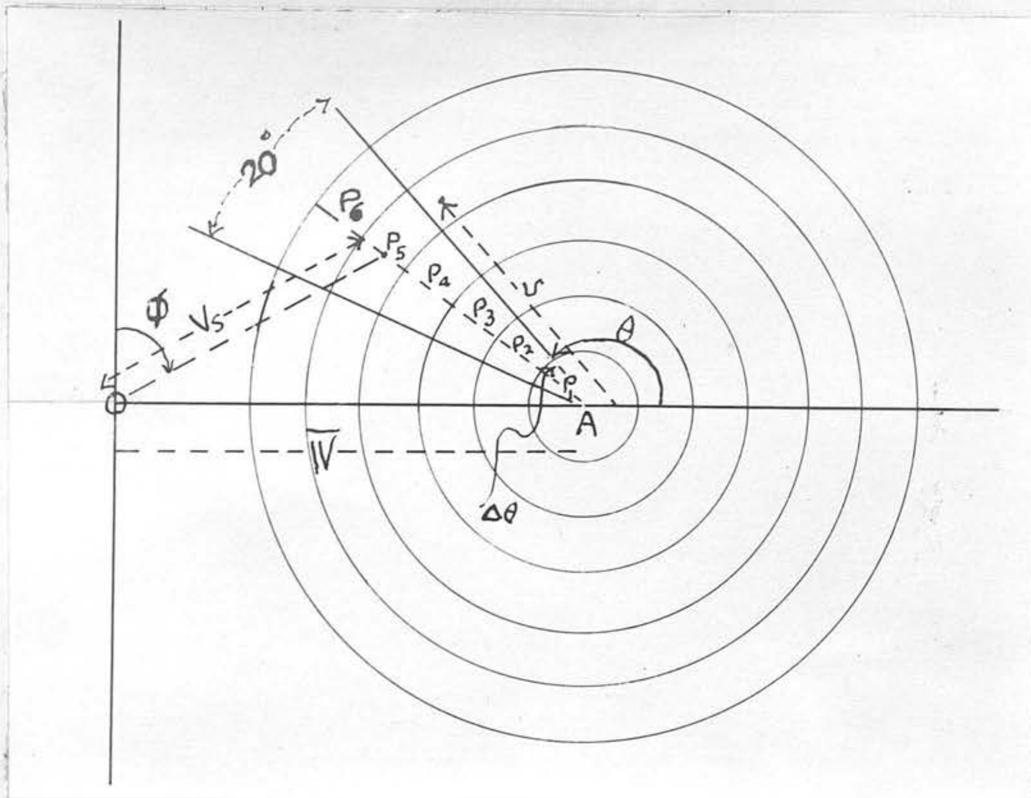


Figure 6(1). Coordinate system for examination of hypothetical distributions.

Let O (Fig.6(1)) be the origin of co-ordinates, and with centre A describe a series of concentric circles with uniformly increasing radii.

Clearly $\qquad OA = \overline{V}$

and if $\qquad OP_n = V(s)$ $\quad$ then

$$\overline{V(s)} = \frac{1}{N}\int_0^{2\pi}\int_0^{v} V(s).\,dv.\,d\theta$$

$$\simeq \frac{1}{N}\sum\sum V(s)\,\Delta v.\,\Delta\theta \qquad \text{........6(1)}$$

where N = total area or number of point-masses within the region bounded by the outermost circle.

For numerical estimation, radii were constructed at convenient equal intervals (20 degrees) and mid-radii such as AP_4 drawn, on which points P_1, P_2..... were located half way between successive boundaries.

In the case examined OA = 15 cms and six concentric circles of radii 1cm, 2cm, 2cm, 3cm, 4cm, 5cm, 6cm, were constructed. O took positions O, O_1, O_2, O_3,.............O_{14}, so that OA = 15 cms, OiA = 14 cms ....$O_{14}A$ = 1 cm. thus giving a range of values for $|\bar{V}|$

Various density distributions were assumed for the disc, the radius of gyration worked out for the particular distribution, the mass computed and a mean value of $V(s)$ obtained by measuring distances such as OP_n as indicated in Fig.6(1). It was realised that the mass centre of each segment does not lie exactly at the mid-point indicated, but this source of error was ignored.

Five density distributions were postulated:

No. 1 Unit masses situated at points P_1, P_2 etc. giving 108 (6 x 18) points in all; the point density is here inversely proportional to the area;

No. 2 Density of the three inner rings ($0 < R < 3$), double that of the outer three rings ($3 < R < 6$);

No. 3 Uniform density;

No. 4 Density of the outer three rings ($3 < R < 6$) double that of the three inner rings;

No. 5 Point masses in each segment of the outer ring only.

It will be appreciated that these five cases correspond to a decreasing degree of concentration at the vector centre, ranging from a type of distribution more concentrated than the "normal" to the extreme U-type distribution of No. 5.

Table 6(1) Values of $q(\%)$ for hypothetical Density Distributions
and (i) Associated Values of $\sigma/|\bar{V}|$
(ii) Value of $\sigma/|\bar{V}|$ appropriate to a Normal Circular Distribution, i.e. $[\sigma/|\bar{V}|]$

OA i.e. $\lvert\bar{V}\rvert$	Hypothetical Distribution														
	No.1			No.2			No.3			No.4			No.5		
	$q(\%)$	$\sigma/\lvert\bar{V}\rvert$	$[\sigma/\lvert\bar{V}\rvert]$	$q(\%)$	$\sigma/\lvert\bar{V}\rvert$	$[\sigma/\lvert\bar{V}\rvert]$	$q(\%)$	$\sigma/\lvert\bar{V}\rvert$	$[\sigma/\lvert\bar{V}\rvert]$	$q(\%)$	$\sigma/\lvert\bar{V}\rvert$	$[\sigma/\lvert\bar{V}\rvert]$	$q(\%)$	$\sigma/\lvert\bar{V}\rvert$	$[\sigma/\lvert\bar{V}\rvert]$
150 mm.	98.7	.230	.235	98.4	.261	.255	98.1	.283	.290	97.9	.298	.299	96.8	.367	.362
140	98.5	.247	.250	98.2	.279	.280	97.8	.303	.300	97.6	.319	.313	96.5	.393	.381
130	98.3	.266	.262	97.9	.301	.299	97.5	.326	.320	97.3	.343	.334	95.9	.423	.409
120	98.0	.288	.288	97.4	.326	.329	97.0	.354	.356	96.7	.372	.370	95.0	.458	.450
110	97.7	.314	.310	97.1	.356	.353	96.5	.386	.381	96.1	.406	.400	94.2	.500	.483
100	97.2	.345	.341	96.3	.391	.390	95.7	.424	.418	95.2	.446	.440	92.6	.550	.546
90	96.5	.384	.381	95.6	.435	.420	94.8	.471	.460	94.3	.496	.480	91.5	.611	.592
80	95.8	.432	.412	94.6	.489	.462	93.7	.530	.508	93.0	.558	.535	89.3	.688	.666
70	94.3	.493	.480	92.8	.559	.543	91.6	.606	.590	90.8	.638	.620	86.4	.786	.755
60	92.2	.575	.562	90.2	.652	.642	88.7	.707	.684	87.5	.744	.720	81.7	.917	.902
50	88.6	.690	.688	85.9	.784	.772	83.7	.848	.840	82.2	.893	.890	74.3	1.10	1.11
40	82.7	.863	.870	78.9	.978	.980	75.9	1.06	1.07	73.9	1.12	1.13	64.2	1.37	1.43
30	72.6	1.15	1.16	67.6	1.30	1.37	63.8	1.41	1.45	61.2	1.49	1.54	50.7	1.83	2.08
20	56.4	1.73	1.74	50.7	1.96	1.98	46.7	2.12	2.20	44.2	2.23	2.34	35.1	2.75	3.07
10	31.7	3.45	3.44	27.2	3.91	4.07	24.6	4.24	4.50	23.0	4.46	4.83	18.1	5.50	6.30

In Table 6(1) are given values of q and of $\sigma/|\bar{V}|$ for the five distributions, together with those for the NCD as estimated from Figs. 5(2) and 5(3) (p.77/8) (certain subsidiary graphical aids were employed to justify the retention of three significant figures).

In general, q is $<$ 70% (approx.) for surface wind distributions and in this range we find, for Nos. 2, 3, 4 and 5 that

$$\left[\sigma/|\bar{V}|\right] > \left(\sigma/|\bar{V}|\right) \text{ hypothetical distributions}$$

the numerical value of the deviation increasing as q decreases, and from distribution No. 2 to No. 5

When q $>$ 85% (approx.) the inequality is almost certainly reversed, but uncertainties arising from the drawing of curves and from graphical estimation do not justify definite statements, but differences are obviously small (considerably less than 1%).

The values for No.1 closely follow those for the NCD, being in deficit (as are the others) in the approximate range 30%, $<$ q $<$ 70% but with indications of an excess as q falls below 30%. The greatest difference in $\sigma/|\bar{V}|$ actually computed arose for distribution No.5, when q = 18.1%, viz. 5.50 compared with the NCD value of 6.30.

It is important to the present thesis to note that only the rather "extreme" type of distribution, i.e. No.5, gives points on the q:$\sigma/|\bar{V}|$ curve outside the "tolerance" limits for the NCD given by the "Handbook" - and even then not throughout the complete range 0% $\leq$ q $\leq$ 100%. Points for distributions Nos. 1 to 4 lie. wholly within the tolerance limits. This is illustrated in Fig.5(3) where points for Nos. 4 and 5 are plotted but not those for Nos. 1, 2 and 3 since, only one a much larger scale diagram would the points be clearly displayed.

As already mentioned, distributions Nos. 1 to 5 represent cases with decreasing central concentration, and the relation of NCD to this sequence is additionally clarified by the trend in the value of the ratio $\sigma/\overline{V(s)}$ for $q = 0 (= |\bar{V}|)$, when we have:-

Distribution	No.1	"NCD"	2	3	4	5
	1.151	1.128	1.087	1.059	1.041	1.000

Clearly in spite of very considerable divergences of the several hypothetical distributions from the normal circular form, deviations from the "normal" $q: \sigma/|\bar{V}|$ relationship are numerically small, and the analysis so far is held to justify a procedure in which the normal circular distribution is considered a valid and useful theoretical model against which to compare actual distributions which diverge markedly from it.

Section 2 The $S/\overline{V(s)}$ ratio

It has been shown (p.80) that

$$S^2/\overline{V(s)}^2 = q^2\left[\left(\frac{\sigma}{|\bar{V}|}\right)^2+1\right]-1 \qquad \text{.........6(2)}$$

and in a limiting case ($q = o$)

$$S^2/\overline{V(s)}^2 = \sigma^2/\overline{V(s)}^2 - 1$$

Values of $S/\overline{V(s)}$ for the several distributions have been computed and are sketched in Fig.6(2) from which the data in Table 6(2) have been derived. In Fig.6 (2) a lighter line has been drawn between the computed value for $q = 0$ and the point corresponding to the lowest value of q employed in Eq.6(2); furthermore values in Table 6(2) are bracketed when $q > 80\%$ or where there is some uncertainty in the drawing of the curves.

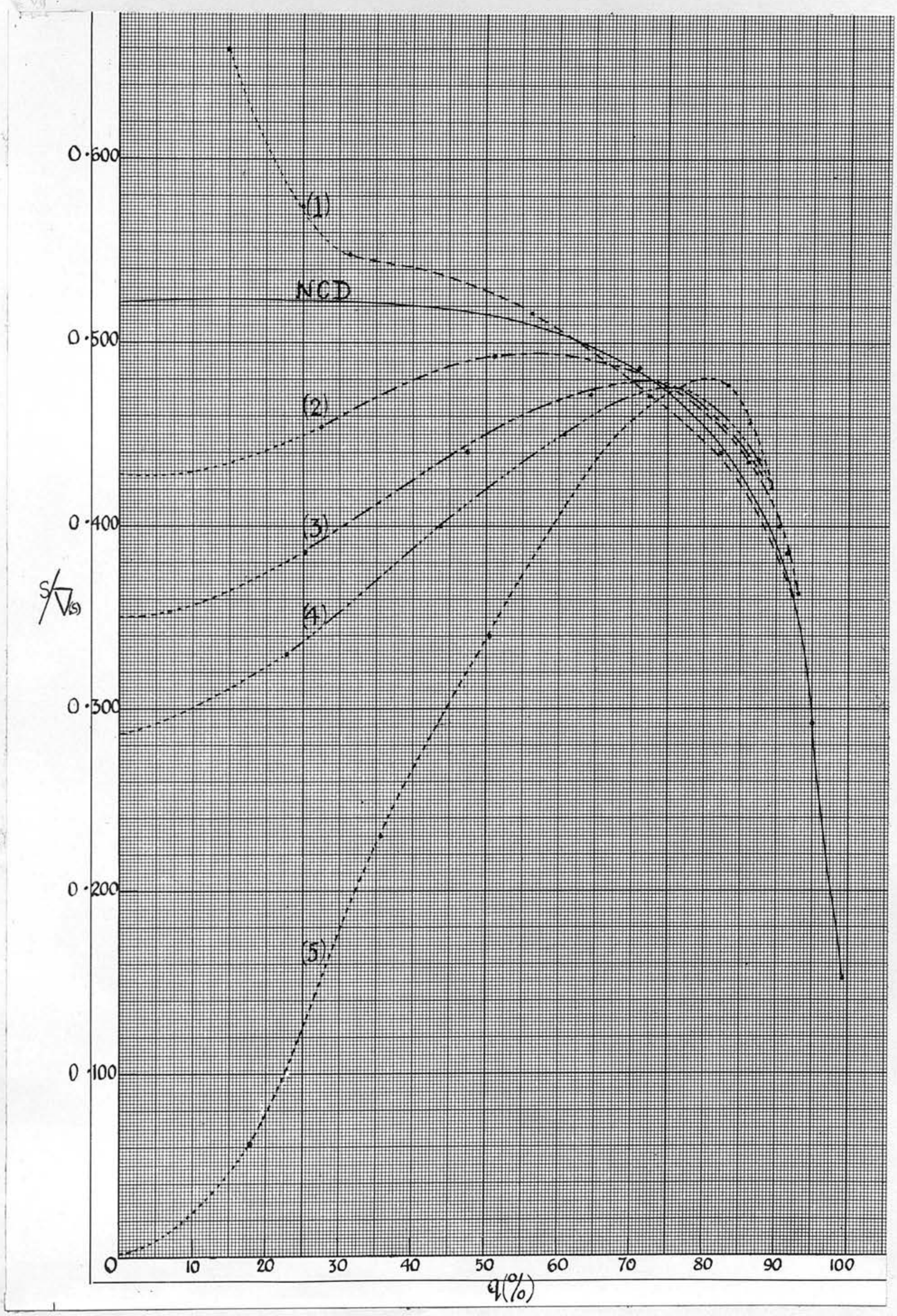


Figure 6(2). To show the relationship between $s/\sqrt{V(s)}$ and q for the normal circular distribution (NCD), and five hypothetical distributions. (see p. 83).

Table 6(2) Values of $S/\sqrt{V(s)}$ against q for the Normal Circular, and five Hypothetical, Distributions

Distribution	q(%) 10	15	20	25	30	40	50	55	60	65	70	75	80	85	90
No.1	(.74)	(.66)	(.61)	(.57)	(.55)	.542	.528	.520	.508	.494	.480	.465	.458	.423	.386
NCD	.523	.522	.522	.522	.522	.520	.516	.512	.506	.498	.488	.476	(.46)	(.43)	(.34)
No.2	(.43)	(.435)	(.44)	(.45)	.459	.480	.492	.494	.493	.490	.485	.476	(.46)	(.44)	(.40)
No.3	(.36)	(.365)	(.375)	.386	.398	.422	.445	.456	.465	.474	.478	.476	(.46)	(.44)	(.42)
No.4	(.295)	(.305)	(.32)	.337	.356	.391	.421	.436	.448	.460	.470	.477	(.47)	(.45)	(.42)
No.5	(.02)	(.025)	.077	.125	.178	.267	.340	.371	.405	.435	.459	.475	.481	(.47)	(.41)

The interpretation of deviation from the value appropriate to the NCD obviously arises, as does the possibility of using such divergencies as a discriminant. Some impression from a necessarily limited experience with the parameter may be noted. Generally $S/\bar{V}_{(s)}$ is less than for the NCD - consistent with some dispersion from the "normal" model, or more probably arising from a bi- or multi- modal directional distribution: possibly also elliptically distributed winds will produce such anomalies. Cases are listed in Appendix II(d) with a relatively low value of q (say $< 30\%$) and of $\bar{V}_{(s)}$ and $|\bar{\mathbf{V}}|$; and a value of $S/\bar{V}_{(s)}$ in excess of the NCD value: this could be associated with the observed fact that a low value of q is rather typical for land stations, where an anomalously high frequency of light wind and calms is to be anticipated. A high concentration of values round the origin, and by supposition near the vector mean, might give the high degree of central concentration typified by the hypothetical distribution No. 1.

Chapter 7

Frequency Distributions, Frequency Density Diagrams, and the Analysis of "Flow" or "Displacement"

Section 1. Frequency distributions

Before discussing the comparison of patterns of actual frequency density and of "flow" with those for a corresponding NCD, it is necessary to examine methods of partitioning into speed and direction ranges, the total frequency for an NCD of given q.

One approach starts with the frequency tables given in "GM 85", extended, insofar as frequencies in speed ranges for all directions ($F(V)$) and directional ranges for all speeds ($F(\psi)$) are concerned, by formulae set out by Knighting (1954) and a tabulation published by the Rand Corporation (1953), (ψ is an angle measured from the vector mean). A second approach, which has not apparently been dealt with explicitly in the literature, is one in which the properties of the frequency density surface are employed directly. The first method involves considerable graphical and numerical interpolation, and whilst the second avoids many of these steps, it does not necessarily follow that it will prove to be the most effective approach for all purposes, though for a quick survey it does seem to provide an efficient tool.

(a) Frequencies according to directional categories.

Knighting (1954) has shown that the total frequency $F(\psi)$ between two values of ψ for all V, which is given by

$$F(\psi) = \frac{1}{\pi\sigma^2}\int_{\psi_1}^{\psi_2} d\psi \int_0^{\infty} exp\left\{-\frac{1}{\sigma^2}\left(V^2 + |\mathbf{\bar{V}}|^2 - 2V.|\mathbf{\bar{V}}|\cos\psi\right)\right\} V.dV$$

may be written

$$F(\psi) = \frac{1}{\pi}\int_{\psi_1}^{\psi_2}\left[\frac{1}{2}\exp(-\alpha^2) + \frac{\sqrt{\pi}}{2}.\alpha\cos\psi \;\; erfc(-\alpha\cos\psi)\exp(-\alpha^2\sin^2\psi)\right] d\psi$$

where $\alpha \equiv |\mathbf{\bar{V}}|/\sigma$

For $\psi_2 - \psi_1$ = 20 degrees (i.e. the interval adopted in "GM 85") he suggests that the approximation

$$F(\psi) = \frac{1}{18}\left[\exp(-\alpha^2) + \alpha\sqrt{\pi}\cos\bar{\psi} \;\; \mathrm{erfc}(-\alpha\cos\bar{\psi}) \exp(-\alpha^2\sin^2\bar{\psi})\right] \qquad 7(1)$$

is sufficient, where

$$2\bar{\psi} = \psi_1 + \psi_2$$

In order to check the accuracy of linear interpolation for obtaining proportionate frequencies where q = 25%, 35% .... 65%, Eq.7(1) was employed; the results are given in Table 7(1).

Table 7(1) Comparison of Relative Frequencies (per mille) for "all speeds" within 20° Sectors from the Direction of the Vector Mean, for Various Values of q: derived (i) from Eq.7(1) (ii) by linear interpolation between Tables xxvi to xxxii ("GM 85").

$\psi_2 - \psi_1$		q					
		15%	25%	35%	45%	55%	65%
0-20°	(i)	69.0	80.2	92.1	107.0	126.4	146.4
	(ii)	69.5	80.1	92.5	107.0	124.5	148.3
20°-40°	(i)	67.0	76.0	86.1	95.7	107.9	118.9
	(ii)	67.3	75.9	85.4	95.7	106.7	119.7
40°-60°	(i)	63.5	69.0	73.9	78.2	81.7	83.1
	(ii)	63.7	68.9	73.7	78.0	81.2	82.7
60°-80°	(i)	59.2	60.7	61.2	60.4	57.7	53.2
	(ii)	59.1	60.6	61.1	60.2	57.7	52.5
80°-100°	(i)	54.6	52.8	50.0	46.0	40.2	33.8
	(ii)	54.3	52.6	49.7	45.9	40.8	33.3
100°-120°	(i)	50.5	46.1	41.2	35.7	28.9	22.6
	(ii)	50.3	46.1	41.1	35.6	29.5	22.3
120°-140°	(i)	47.3	41.1	35.1	28.9	22.2	16.1
	(ii)	47.1	41.3	35.3	29.1	22.9	16.3
140°-160°	(i)	45.0	37.9	31.3	25.0	18.5	12.8
	(ii)	44.9	38.1	31.5	25.2	19.2	13.3
160°-180°	(i)	43.8	36.3	29.2	23.2	17.3	11.9
	(ii)	43.8	36.6	29.7	23.4	17.5	11.7

The agreement is excellent and checks not only the basic "GM 85" estimates but also the q : $\sigma/|\bar{V}|$ relationship previously obtained (page 74), which is required for estimating the value of $\alpha \left(\equiv |\bar{V}|/\sigma\right)$ in Eq.7(1). It does not necessarily follow that the linear interpolation would be equally good at all stage from $V = 0$ to $V = V$. The angle ψ and the interval $\psi_2 - \psi_1$ are at our disposal, and limits may be chosen making it possible to partition frequencies in directional ranges centred upon any selected compass directions.

(b) Frequencies according to speed ranges.

Knighting (1954) also gives for F(V), i.e. the frequencies for all directions between speeds 0 and V, the expression

$$F(V) = \frac{2}{\sigma^2}\int_0^V \exp\left\{-\frac{V^2+|\bar{V}|^2}{\sigma^2}\right\} I_0\left(\frac{2\,V.|\bar{V}|}{\sigma^2}\right) V.dV$$

which, when $V^2/\sigma^2 \equiv \omega^2$ and $|\bar{V}|/\sigma \equiv \alpha$ reduces to

$$F(V) = \exp(-\alpha^2)\sum_{n=0}^{\infty}\frac{\alpha^{2n}}{(\underline{|n})^2}\int_0^{V^2/\sigma^2} \exp(-\omega)\,\omega^n\,.\,d\omega \qquad \text{.......7(2)}$$

In the earlier work in the present investigation, interpolations were checked by using the above equation, later the Rand tables (see Appendix II(a)) came to hand and these were used instead.

A comparison between some estimates of F(v) obtained from (i) Rand Tables, (ii) Equation 7(2) and (iii) "GM 85" are given in Table 7(2) on page p.96, together with graphically estimated values (iv) and (v) from Figs.7(1) and 7(2) It may, however, be stated here that the difference between (1) - presumed the correct value - and the others are negligible: the differences being only of the order of a few parts in a thousand; accordingly earlier results obtained using Eq.7(2) did not require revision.

(c) Frequencies according to direction and speed.

Two methods were used; one - to be described now - based upon the "GM 85" Tables and upon graphical interpolation, the second - dealt with in the section to follow (p. 97) - in which a direct computation was employed.

In the first method two families of curves were constructed:-

(i) Fig.7(1): in which for each 20° directional category, e.g. $020^\circ - 040^\circ$, curves were drawn of cumulative frequency against ascending values of $V/|\bar{V}|$ for each of q = 10%, 20%, 30% .... 70%; (these curves are termed Type I curves).

(ii) Fig.7(2): in which frequencies from $V/|\bar{V}| = 0$ to stated values of $V/|\bar{V}|$ were plotted against q for each of the direction ranges; (these curves are designated Type II curves).

Fig.7(1) is thus one of a series of graphs, each of which applies to a particular range of ($\psi_2 - \psi_1$), whilst Fig.7(2) is one of a series, each of which corresponds to a particular terminal value of $V/|\bar{V}|$.

Cumulative frequencies for non-decadal values of q, e.g. q = 35% up to $V/|\bar{V}| = 3$ are thus given either by f_1, f_2...... from Fig.7(1) and the associated graphs, or by the sum of increments F_1, F_2, from Fig.7(2).

If frequencies for certain integral values of $V/|\bar{V}|$ are required, the appropriate Type II may be used directly, but the main purpose of this set is to provide points such as P in Fig.7(1) which will assist in the construction of a Type I curve, of which (xy) is a portion, for non-decadal values of q. Type II curves are of least help when q is less than about 30% as the isopleths become crowded, and slight errors in drawing are relatively serious. In practice, Type I curves for small q were built up using the Rand Tables after obtaining, from Fig.5(2) or Fig.5(3), the appropriate value of $|\bar{V}|/\sigma$. The accuracy of various estimates of F(V) for certain q and $V/|\bar{V}|$ may be gleaned from Table 7(2). The scales adopted for the various graphs were:-

for Type I curves - abscissa; $n(\equiv V/|\bar{V}|)$, $1 \equiv 1$ in

ordinate; frequency per mille, 10 parts $\equiv 1$ in. or 2 in.

for Type II curves - abscissa; q, 10% $\equiv 1$ in.

ordinate; frequency per mille, 10 parts $\equiv 1$ in

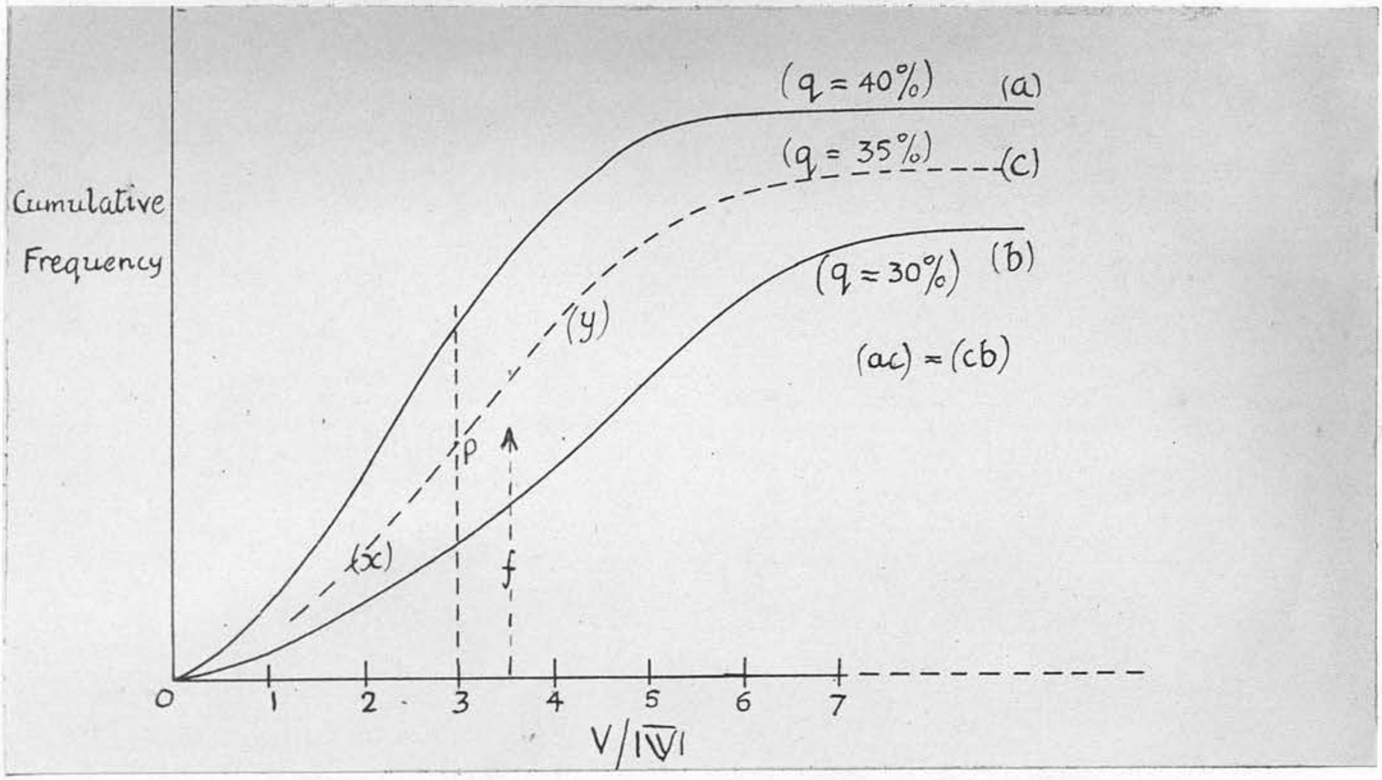


Figure 7(1). Schematic diagram to show general relationships between cumulative frequency and $V/|\bar{V}|$ for various q and a particular directional range $(\psi_1-\psi_2)$. The Type I curve. (see p. 93).

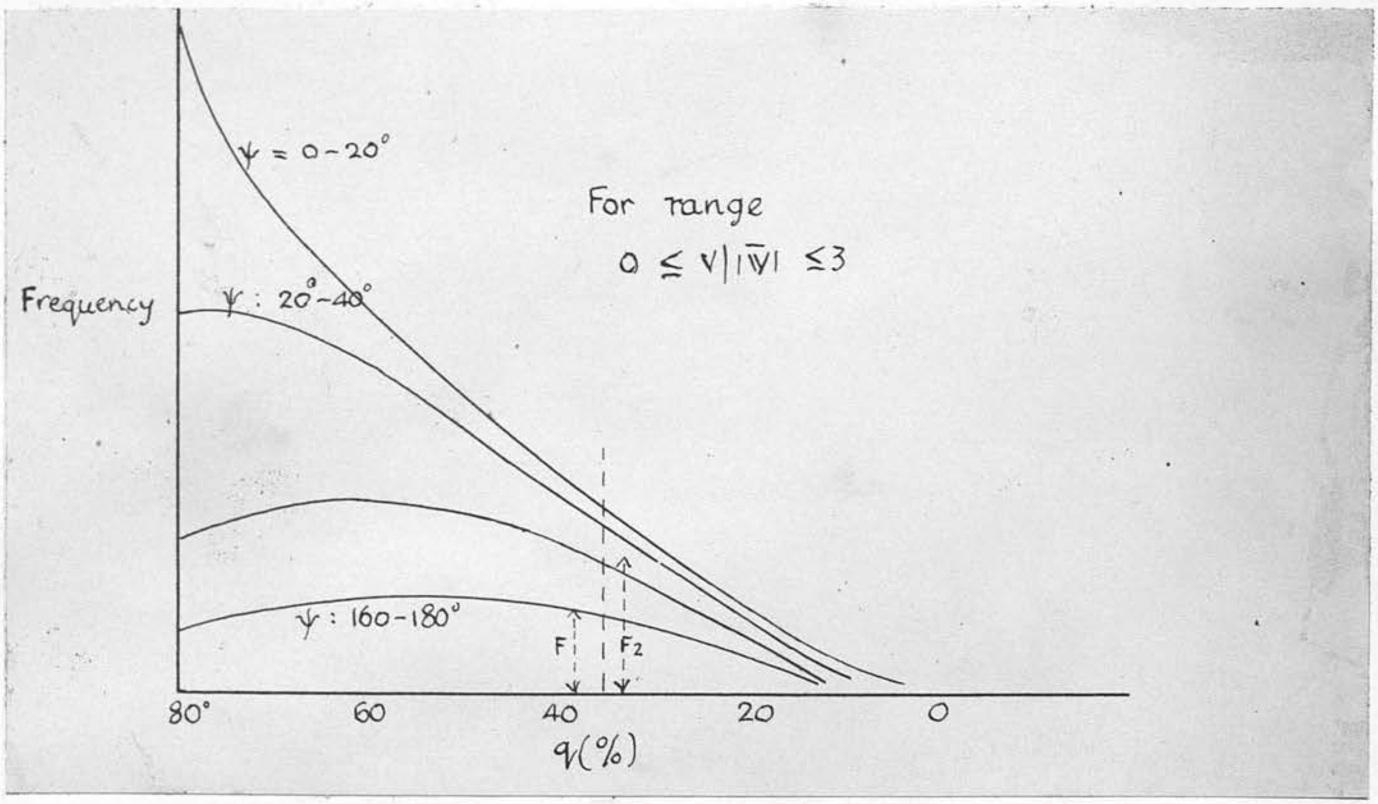


Figure 7(2). Schematic diagram to show general relationships between frequencies in various directional ranges, for given q and range of $V/|\bar{V}|$. The Type II curve (see p. 93).

Table 7(2) Estimates of Relative Cumulative Frequency for Speed Intervals 0 to n ($\equiv V/|\bar{V}|$) irrespective of direction; for Normal Circular Distributions of stated q, assuming N = 1000 and $|\bar{V}| = 1$:
(i) according to the Rand Tables (the correct value);
(ii) according to Equation 7(2);
(iii) directly from, or, for q = 35% and q = 55%, by linear interpolation from, Tables in "GM 85";
(iv) from graphs of Type I;
(v) from graphs of Type II.

q%	Method	$n \equiv V/\lvert\bar{V}\rvert$ 1	2	3	4	6
20	(i)	31	117	241	387	667
	(ii)	30	115	240	386	666
	(iii)	30	115	241	388	669
	(iv)	31	115	241	388	671
	(v)	30	115	241	387	668
30	(i)	–	–	460	–	916
	(ii)	–	–	461	–	916
	(iii)	–	–	463	–	918
	(iv)	–	–	463	–	918
	(v)	–	–	463		917
40	(i)	–	–	674	–	–
	(ii)	–	–	673	–	–
	(iii)	–	–	678	–	–
	(iv)	–	–	677	–	–
	(v)	–	–	676	–	–
50	(i)	–	534	–	957	–
	(ii)	–	536	–	960	–
	(iii)	–	538	–	959	–
	(iv)	–	538	–	958	–
	(v)	–	538	–	958	–
35	(i)	–	–	575	–	969
	(ii)	–	–	577	–	968
	(iii)	–	–	571	–	954
	(iv)	–	–	570	–	954
	(v)	–	–	574	–	966
55	(i)	–	615	–	982	–
	(ii)	–	617	–	979	–
	(iii)	–	605	–	975	–
	(iv)	–	605	–	975	–
	(v)	–	609	–	977	–

Clearly, therefore, errors which arise from the construction of graphs for decadal values of q, and from the reading of values from the relevant graphs are, up to this stage, negligible. The magnitude of the errors incurred by estimating from interpolatory curves such as (xq) in Fig.7(1) constructed for intermediate values of q, may be judged by comparing estimates (iii), (iv) and (v) against (i) and (ii) for q = 35% and q = 55%; additional errors may arise in the further graphical steps required for the construction of "flow" diagrams (see p. 104).

A frequency table may thus be obtained in which directional groups are defined in 20 degree intervals from the direction of the vector mean and speed intervals in relation to $V/|\bar{V}|$, where V is the upper limit of a class interval used in the original extraction. Frequencies then need to be partitioned such that the mid-radii of the new 30° directional ranges coincide with the fixed directions, i.e. 0°, 30°, 60°, etc. used in the basic analyses.

Section 2. Frequency density diagrams

Suppose, Fig.7(3), the two-dimensional field be divided into segments defined by $v, v+\Delta v;\ \theta, \theta+\Delta\theta$ and that $\alpha_1\ \alpha_2\ \alpha_3 \ldots$ are the areas of the radially symmetrical small segments; then if $n_1, n_2, n_3, \ldots$ vector ends fall respectively within these areas, the frequency densities are proportional to

$$\frac{n_1}{\alpha_1} \qquad \frac{n_2}{\alpha_2} \qquad \frac{n_3}{\alpha_3}$$

If N be the total number of observations in a NCD of standard vector deviation σ and vector mean $\bar{V}$ then the frequency density at a distance represented by v from the vector mean is proportional to z

where

$$z = \frac{N}{\pi\sigma^2} \exp\left(-v^2/\sigma^2\right) \qquad \ldots\ldots\ldots\ldots 7(3)$$

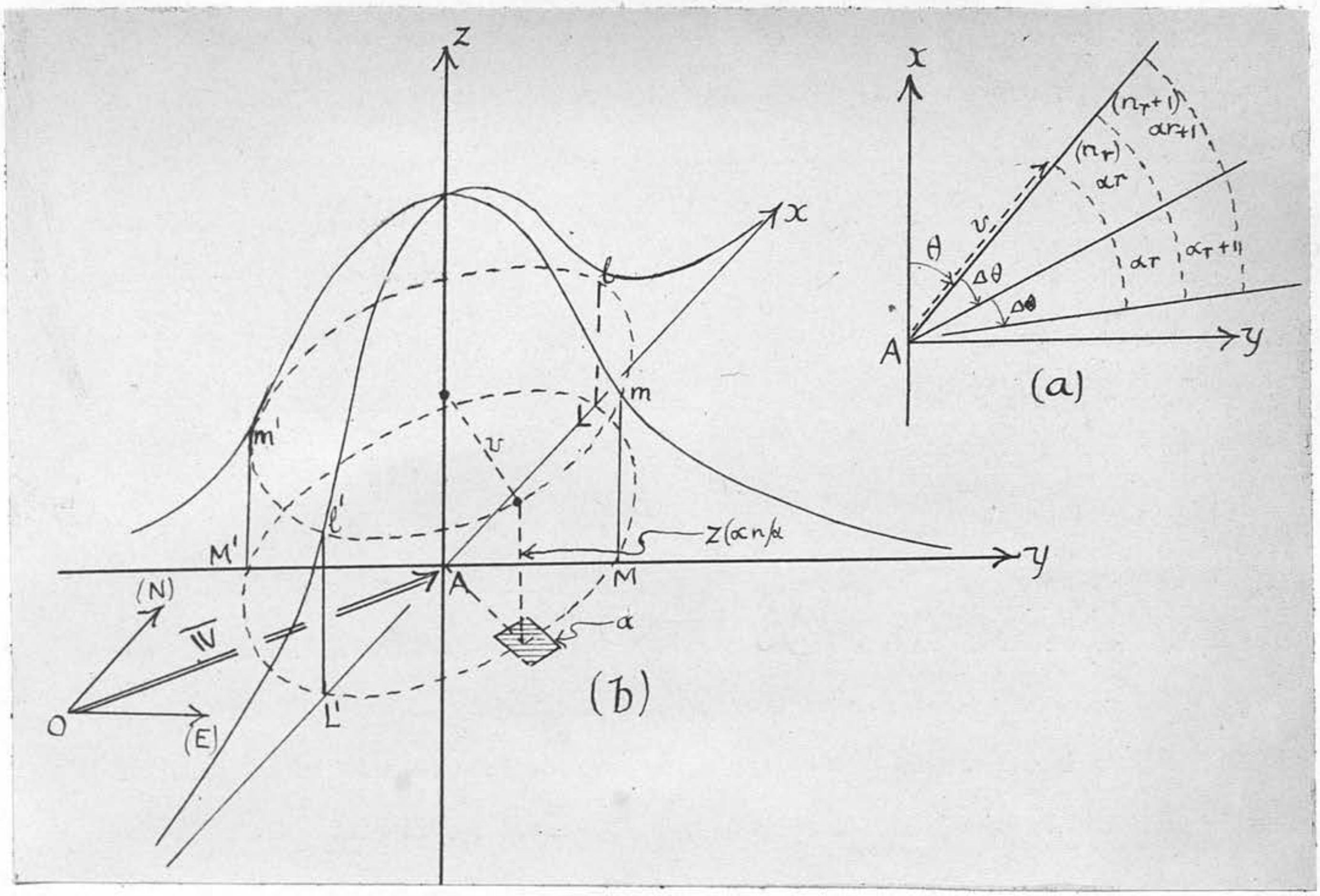


Fig. 7(3). Schematic Diagrams to show:

(a) division of two dimensional field into segments symmetrical about A, the end of the vector mean;

(b) the geometry of the frequency density surface.

For graphical comparisons of frequency distributions, isopleths of frequency density corresponding to convenient values of z, such as z = 0.1, 0.2, 0.3, etc. have been used, the appropriate value of v being given by

$$v^2/\sigma^2 = -\ln.\left(z\,\frac{\pi\sigma^2}{N}\right)$$

Circular isopleths so defined may then be compared with the pattern constructed from the actual data. Possible anomalies are immediately evident and may be additionally specified by the construction of anomaly patterns using an ordinary gridding technique. These anomaly patterns may then be examined against the topographical and other physical factors likely to influence the wind distribution.

The further possibility of analysing anomalies numerically by a comparison of actual and theoretical frequencies presents itself, but this latter method, whilst more suitable for statistical tests, is not sufficient for a full physical analysis.

Construction of Theoretical Frequency Table by the Direct Use of Equation 7(3)

Consider a theoretical distribution, vector mean $\overline{V}$ at an angle ϕ from North. Suppose it is required to estimate the theoretical frequency within a segment such as (a b c d) of area α (Fig.7(4)

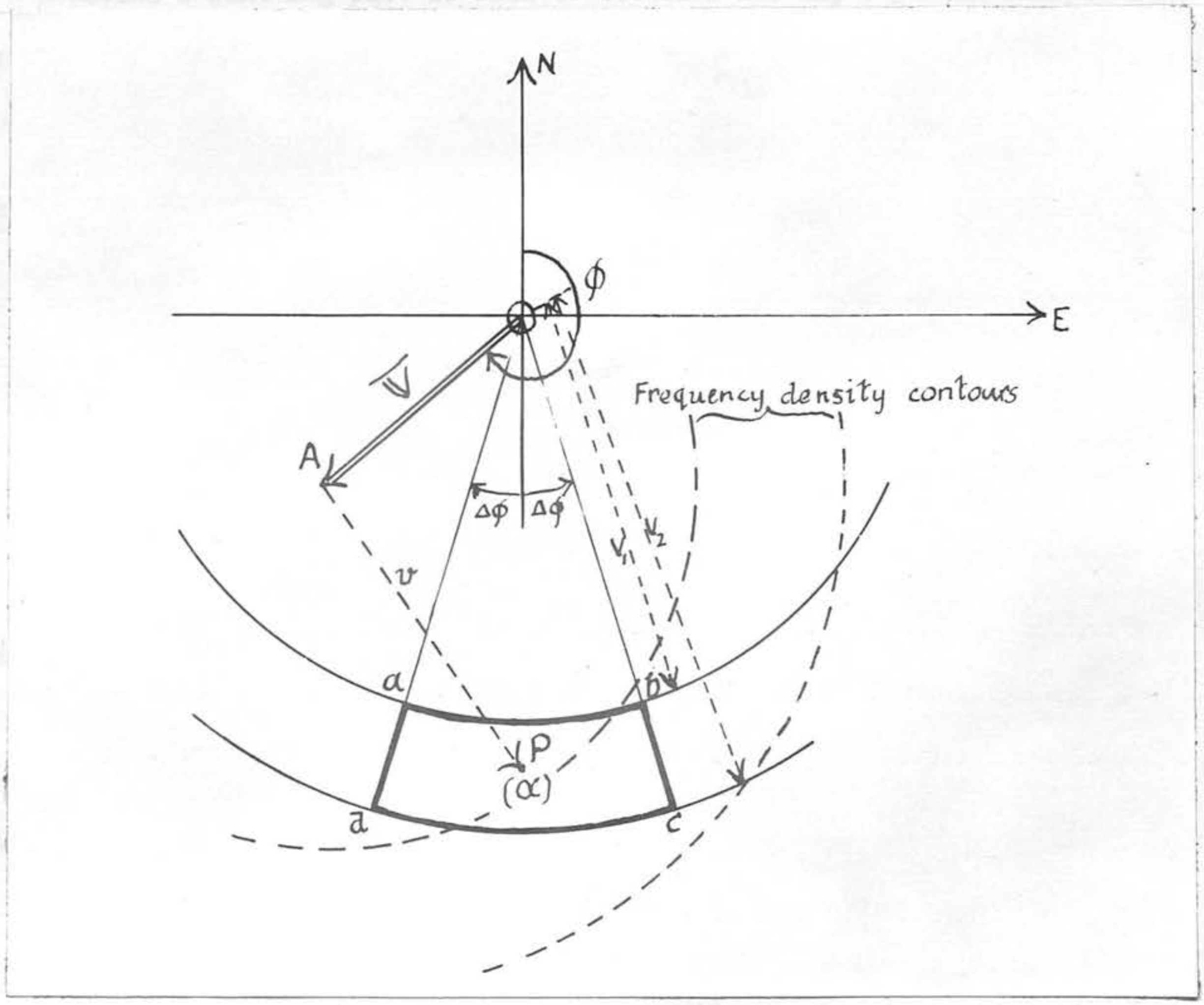


Fig. 7(4). Schematic diagram to illustrate direct computation of frequencies from the frequency density surface.

Assuming the frequency to be uniformly distributed within (abcd), and that it may be centred with sufficient accuracy at a point P distant $\frac{1}{2}$ (V + V) from O, then given $\overline{\mathbb{V}}$, q (thus σ), N, and the distance v , (this latter could be calculated but in practice it seems sufficient to measure on a polar diagram), we may find z from

$$\frac{v^2}{\sigma^2} = -\ln.\left(z.\frac{\pi\sigma^2}{N}\right)$$

and the required frequency

$$= z \text{ x area (abcd)}$$

In practice a semi-log plot is used to obtain z for any v . An example is given in detail later (p. 106) after the construction of flow diagrams has been discussed.

Section 3. The analysis of "flow" or "displacement".

(a) Total Flow.

Let $[D]$ $\equiv$ the total displacement or flow past any fixed point within some stated period;

and $D(V\phi)$ $\equiv$ the contribution to $[D]$ from winds within a cell defined by $V, V+\Delta V;\quad \phi, \phi+\Delta\phi$

then $[D] \equiv \sum_{\phi}\sum_{V} D(V,\phi)$

Also, if f_1 be the frequency of winds within a particular cell where the scalar mean speed is $\overline{V(s)}_1$, then

$$\sum_{V}\sum_{\phi}\left(f_1 \overline{V(s)}_1\right) \equiv \sum_{\phi}\sum_{V} D(V,\phi) \equiv N.\overline{V(s)}$$

but since q = $|\overline{\mathbb{V}}|/\overline{V(s)}$

then

$$q \equiv N|\overline{\mathbb{V}}|/N.\overline{V(s)}$$

$$\equiv N|\overline{\mathbb{V}}|/D$$

and

$$[D] \equiv \frac{1}{q}N.|\overline{\mathbb{V}}| \qquad \ldots 7(4)$$

and if

$$|\overline{\mathbb{V}}| = 1$$

$$[D] = N/q \qquad \ldots 7(5)$$

Eq.7(4) is an identity independent of the actual character of the wind distribution, but subject, in actual numerical work, to small errors arising in computational processes such as grouping, and the associated assumption that the mean speed for a group lies at the mid-range of the speeds.

The evaluation of [D] for the NCD is also required. This may readily be done using Tables XXI to XXII in "GM 85" - results are set out in Table 7(3).

Table 7(3) Comparison of Estimates of Total Displacement [D] for various q, $N = 1000$ and $|\bar{V}| = 1$
(i) from the identity Eq.7(5)
(ii) as derived from Tables XXI to XXII ("GM 85")

q	Total Displacement [D] (i)	Total Displacement [D] (ii)
10%	10000	10041
20%	5000	5055
30%	3333	3367
40%	2500	2496
50%	2000	2009
60%	1667	1675
70%	1429	1431
80%	1250	1244
90%	1111	1108
95%	1053	1056

The only differences worth mentioning are the slight (c.1%) over-estimates at $q \leqslant 30\%$, part of which may well be due to the assignment of the mid-range of an interval as the mean value for that interval. Eq.7(4) provides an immediate check on successive graphical and other approximations involved in constructing "flow" diagrams for an NCD of given $|\bar{V}|$ and N, and having a value of q lying between the values quoted in Table 7(3).

(b) Partitioning of "flow" according to speed ranges.

This can be done conveniently by computing marginal frequencies for "all directions" and multiplying by the mid-value of the speed interval concerned.

(c) Partitioning of "flow" according to directional ranges.

It was found convenient to compute the "flow" for each "cell" in the "GM 85" Tables and the total for "all speeds" within a given directional category; - results are given in Table 7(4).

Table 7(4). Flow or Displacement for a Normal Circular Distribution, for a given " q ", and assuming N = 1000 and $|\bar{V}| = 1$.; in 20 Intervals from the Vector Mean. (Derived from "GM 85", Tables XXII to XXXII).

q (%)	Angular interval from vector mean (degrees) 0–20	20–40	40–60	60–80	80–100	100–120	120–140	140–160	160–180	Total (a)
10	676.7	658.0	626.8	591.6	550.5	514.8	486.4	462.4	453.1	10,041.6
20	404.9	381.7	350.0	307.5	266.3	233.3	208.4	191.9	183.7	5,055.0
30	318.4	293.7	251.9	207.6	167.5	136.0	114.5	99.9	94.0	3,366.7
40	278.8	247.7	200.3	151.4	111.6	83.6	67.2	56.1	51.4	2,496.0
50	262.0	224.4	167.7	115.8	79.1	54.3	39.9	32.2	29.2	2,009.1
60	257.9	207.5	141.7	88.1	53.1	33.5	22.9	17.7	15.0	1,675.2
70	264.6	197.4	117.9	61.4	37.8	17.4	10.8	8.1	6.5	1,431.2
80	284.8	185.3	87.2	35.4	14.3	6.6	3.7	2.3	2.1	1,243.7
90	335.8	158.4	44.5	10.3	2.8	1.0	0.5	0.3	0.2	1,107.5
95	396.0	115.1	15.1	1.5	0.2	0.04	0.01	0.01	-	1,056.0

(a) Total for the range 0° to 180° and 0° to -180° from the vector mean. This total independently derived and hence not exactly twice the sum of the individual entries.

For interpolation, the foregoing data were graphically displayed with D against either angular intervals as argument, or against q as argument, resulting in a family of curves in q or $(\psi_2 - \psi_1)$ respectively. The latter method was found to be more convenient - especially for interpolation between given values of q - and in practice three of the possible systems of axes were used:

viz. D against q

log D against q

log D against log q

Each of the three methods demonstrated some superiority over the others as regards estimation at different points of the range and the mean (or more generally a reasonable modal) value adopted. Differences between estimates were typically less than 1%. This method of constructing the two-dimensional flow field for an NCD having the same parameters as an actual distribution thus involves the following steps:

(i) The construction of a frequency table with directional intervals of 20 degrees and speed intervals $V_1/|\bar{V}|$, $V_2/|\bar{V}|$ ... where $V_1, V_2, \ldots\ldots\ldots$ are the limits of the speed class used in the original extraction, (graphs of Type I and/or Type II illustrated in Figs. 7(1) and 7(2) are employed);

(ii) Multiplying each cell frequency by a mean speed for the cell, so obtaining the contribution $D(V,\phi)$. A check on gross errors at this stage is given by the flow for "all speeds" derived from Table 7(4), and the graphical interpolations obtained therefrom. Small errors were found to arise at this stage mainly due to errors in constructing Type I curves for intermediate q .

(iii) By means of a cumulative graph, the flow for each speed stratum is then repartitioned graphically into direction groups from the vector mean.

For example, suppose, Fig. 7(5), the vector mean makes angle α with west, then we require an estimate of the total flow for the NCD which would occur within a segment such as (abcd). This is clearly given by the intercepts from a cumulative graph, at angles (α - 15) and (α + 15), etc.

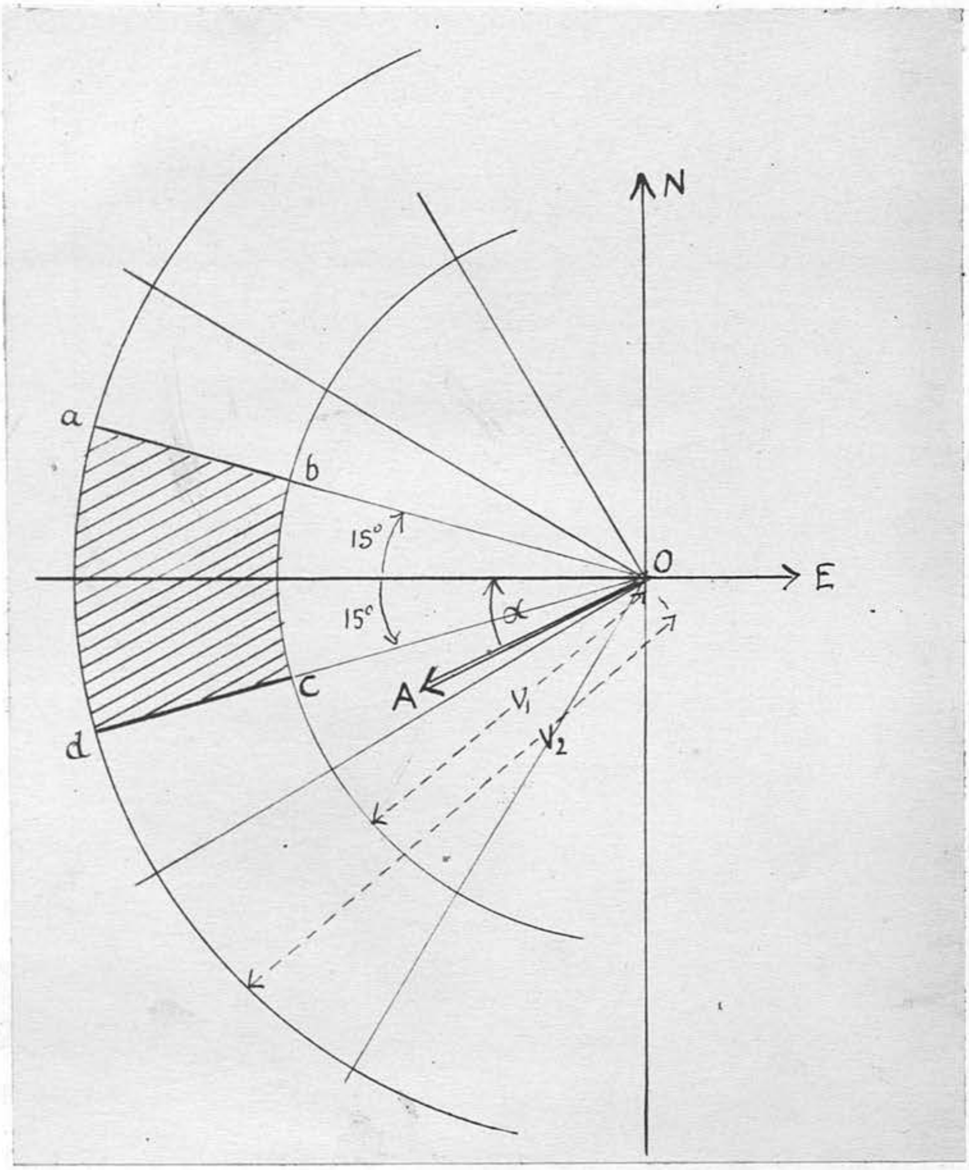


Figure 7(5). Schematic diagram to illustrate partitioning of displacement on the basis of the standard 30° intervals (viz. 345° - 015°, 015° - 045° etc.).

The alternative method, based on the direct use of the frequency density equation, is sufficiently indicated by the consideration of Fig. 7(4).

Both these methods were used in analysing the data, and the comparisons given in detail in Tables 7(5) to 7(8) show the measure of agreement achieved.

Section 4. Examples of analyses of wind observations by various methods.

To illustrate the methods already discussed, two examples will be considered in some detail.

In the first a comparison has been made of frequencies from an analysis given in "GM 85" (p. 6 et seq) of a group of upper winds - in which a numerical partitioning into speed and direction categories was adopted - and those obtained using the properties of the frequency density surface directly: furthermore, marginal totals following other methods are included. In the second example, a series of 896 observations of surface winds at Bell Rock has been analysed, quantities for the appropriate NCD having been obtained by means of the graphical technique described earlier, and by the frequency density method. In addition, for both series, estimates of displacement are given, whilst for the second, diagrams illustrating the results are presented.

Example 1.

Results are set out in Table 7(5). The method used by Brooks et al (1950) is fully explained in their treatise, and that used by the writer in the previous sections. The wind distribution discussed in "GM 85" has a value for q of 50%, which, according to Brooks et al. implies a value of 2.04 for $\sigma/|\bar{\mathbf{V}}|$ compared with 2.00 indicated by Fig. 5(2). In order to ensure a more direct comparison with the numerically derived values of Brooks, the former estimate was adopted when obtaining marginal totals by the "Rand" tabulations.

Table 7(5). Frequencies (per mille) for a Normal Circular Distribution, given

$|\bar{\mathbf{V}}| = 16.5$ knots. $\phi = 283^{\circ}$ $q = 50\%$

as derived: (i) by Brooks et al, "GM 85"
(ii) from frequency densities (i.e. Equation 7(3)) together with marginal totals for "all speeds" and "all directions",

and including: (K) estimate for "all speeds" (Knighting (1954))
(R) estimate for "all directions" from the Rand Tables (see Appendix II(a)).

Directional Interval and Method		0 - 3	3 - 14	14 - 27	27 - 41	41 - 54	54 - 68	68 - 81	81 - 95	95	"All speeds" (i)	"All speeds" (ii)	"All speeds" (K)
N	(i)	-	14.8	36.2	37.0	21.3	9.0	2.5	0.5	..	121.3		117.5
	(ii)	0.8	16.8	36.0	37.4	19.9	8.0	2.1	0.5	-		121.5	
NE	(i)	-	11.9	22.8	17.5	7.4	2.3	0.4	0.1	..	62.4		62.3
	(ii)	0.8	13.4	23.2	17.6	6.8	2.2	0.4	..	-		64.4	
E	(i)	-	10.5	18.0	11.4	4.1	1.3	0.3	..	..	45.6		45.1
	(ii)	0.7	12.1	17.8	11.6	3.9	1.0	0.1	..	-		47.2	
SE	(i)	-	10.9	19.4	13.2	4.8	1.4	0.2	..	..	49.9		49.4
	(ii)	0.7	12.4	19.0	13.1	4.8	1.3	0.2	..	-		51.5	
S	(i)	-	13.0	27.6	23.8	11.6	4.1	0.9	0.3	..	81.3		80.1
	(ii)	0.9	14.5	28.3	23.9	10.2	4.0	1.0	0.2	-		83.0	
SW	(i)	-	16.1	43.6	49.7	32.1	15.3	4.6	1.1	0.2	162.7		160.8
	(ii)	0.8	17.3	44.2	50.5	30.1	15.4	4.1	1.1	-		163.5	
W	(i)	-	18.1	56.2	73.3	54.7	29.8	10.4	2.8	0.4	245.7		254.4
	(ii)	0.8	19.3	56.9	77.0	56.7	31.5	10.6	3.1	-		255.9	
NW	(i)	-	17.3	52.0	65.1	46.6	24.4	8.1	2.2	0.4	216.1		224.5
	(ii)	0.8	18.7	47.3	68.4	45.1	24.8	7.6	2.1	-		214.8	
	(i)	15.0(a)	112.6	275.8	291.0	182.6	87.6	27.4	7.0	1.0	1000.0(b)		1000.0
	(ii)	6.3	124.5	272.7	299.5	177,5	88.2	26.1	7.0	-		1001.8(c)	
	(R)	8	117	276	295	179	90	26	8	1	1000		

(a) Total frequency in this group not partitioned by direction in "GM 85".
(b) Including 15.0 for lowest speed group.
(c) Difference between this value and the correct value of 1000.0 indicates extent of error induced by using the step-by-step approach to the total.

Brooks et al. in "GM 85" did not attempt a directional partitioning of the frequencies in the 0 - 3 knot interval, but did estimate frequencies for speeds > 95 knots. The writer, however, has obtained an estimate for the lowest speed category, but not one for the final, open, interval, as this depends on an arbitrary decision regarding an appropriate mean speed; even so, the adoption of some plausible value, viz. a knot or so in excess of 95 knots, could only result in an addition to the total frequency of one or two parts in a thousand. It will be noted that, as far as marginal totals are concerned, there is, on the one hand, agreement to within a few percent as between methods (i) and (ii), and on the other hand equally good agreement between the results from these methods and the independent checks afforded by Knighting's formula and the "Rand" tabulation. Agreement within individual cells must, it is suggested, also be regarded as satisfactory.

Example 2.

The data analysed are some 896 observations of hourly wind speeds obtained twice daily (for hours ending 0400 and 1600GMT) at Bell Rock Lighthouse for the months of September, October, November 1951 - 55 inclusive (observations were missing for seven days in September, 1955). Method designated (i) in Table 7(6) is based upon Eq. 7(3); that designated (ii) on the value of $D(V,\phi)/\bar{V}$ for each cell, $D(V,\phi)$ having been obtained by the procedures described in Section 3 of this chapter.

In this analysis (as in all others undertaken in this investigation) the standard twelve direction groups are used, viz. $345^\circ - 015^\circ$, $015^\circ - 045^\circ$, etc. The class interval for speeds, and the relative values of the areas (α) of the various segments required for the frequency computation (see Fig. 7(4)), are given below:

Class Interval (knots)	< ½	½ - 3½	3½-6½	6½-12½	12½-18½	18½-24½	24½-30½	30½-36½	36½-42½	42½-48½
Area α (arbitrary units)		3.206	7.85	29.83	48.68	65.52	86.36	105.20	124.05	142.89
$\frac{1}{\alpha}$ ($\times 10^3$)		311.9	127.4	33.5	20.5	14.8	11.6	9.5	8.1	7.0
		90.02								

As mentioned earlier (p. 63) we are, in general, faced with a decision as to which of several parameters should be regarded as defining the NCD. However, for the series under discussion differences appear to be small, viz.

$$q = 42.7\% \text{ and hence } \left[\sigma/|\bar{M}|\right] = 2.436$$

compared with the actual value $\quad (\sigma/|\bar{M}|) = 2.42$

Although the graphical and other interpolatory steps used hardly justify a decision that such a small difference should be granted significance, yet, as will be seen from a comparison of the last two rows in Table 7(6), the use of the slightly different alternative values of give rise to numerical differences which may prove to be significant: this topic is further considered in Chapter 9.

Table 7(6). Frequencies for a Normal Circular Distribution, given:

$N = 896$ $|\bar{V}| = 7.49$ knots $\phi = 257$ degrees $q = 42.7\%$ and $\sigma/|\bar{V}| = 2.42$

as derived: (i) from the frequency density surface (Eq. 7(3))
(ii) from "displacement" $D(v,\phi)$,

together with: [R] marginal totals for "all directions" (Rand Tables) based on $[\sigma/|\bar{V}|]$
(R) as for [R], but based on $(\sigma/|\bar{V}|)$

Directional Category and Method.		Speed Categories (speeds in knots). ½ - 6½	6½-12½	12½-18½	18½-24½	24½-30½	30½-36½	36½-42½	42½-48½	All speeds (i)	All speeds (ii)
360°	(i)	7.4	15.2	14.8	9.7	4.6	1.8	0.5	..	54.0	
	(ii)	7.0	14.6	13.6	10.4	5.5	1.7	0.7	..		53.5
030°	(i)	7.1	12.5	10.5	6.1	2.8	0.8	0.2	..	40.0	
	(ii)	6.9	12.2	10.5	6.3	2.8	1.0	0.5	..		40.2
060°	(i)	6.6	11.0	8.8	4.8	1.0	0.5	0.1	..	32.8	
	(ii)	6.5	10.8	9.1	4.5	2.1	0.7	0.2	..		33.9
090°	(i)	6.6	11.0	8.8	4.8	1.0	0.5	0.1	..	32.8	
	(ii)	6.3	11.1	8.9	4.3	2.1	0.6	0.1	..		33.4
120°	(i)	7.1	12.4	10.4	5.9	2.6	0.8	0.1	..	39.3	
	(ii)	7.2	12.1	10.4	6.3	2.6	1.0	0.4	..		40.0
150°	(i)	7.6	15.0	14.1	9.2	4.5	1.6	0.5	..	52.5	
	(ii)	7.7	14.1	12.9	9.4	5.2	1.6	0.7	..		51.6
180°	(i)	8.2	18.6	20.9	15.5	8.6	3.7	1.4	0.3	77.2	
	(ii)	7.4	17.3	19.1	15.5	8.8	3.5	1.7	0.3		73.6
210°	(i)	8.9	22.5	28.2	24.1	15.9	6.9	2.7	0.9	110.1	
	(ii)	9.0	22.3	26.4	23.0	15.5	7.3	2.9	1.1		107.5
240°	(i)	9.2	24.5	34.1	30.8	21.6	11.0	4.5	1.4	138.0	
	(ii)	10.1	23.8	33.7	28.8	20.2	11.2	4.6	1.5		133.9
270°	(i)	9.3	25.3	34.6	32.1	21.6	11.0	4.6	1.4	139.9	
	(ii)	9.7	23.7	34.6	29.9	20.8	12.4	4.8	1.6		137.5
300°	(i)	8.7	23.2	29.0	24.3	15.9	10.7	2.9	0.9	115.6	
	(ii)	9.1	22.2	27.3	24.3	16.3	8.0	3.1	1.2		111.5
330°	(i)	8.2	15.2	21.6	16.1	8.6	3.9	1.4	0.1	75.1	
	(ii)	8.1	17.7	20.1	16.0	9.7	3.8	1.8	0.4		77.6
All directions.	(i)	94.9	207.2	235.8	183.4	108.7	53.2	19.0	5.0	907.3	
	(ii)	95.0	201.9	226.6	178.7	111.6	52.8	21.5	6.1		894.2
	[R]	93.2	200.7	227.6	181.9	110.2	52.8	20.6	8.0	896	
	(R)	97.7	199.9	227.5	181.8	110.2	52.0	19.7	7.2	896	

The total frequency obtained by method (i) is 907.3, or a little more than one percent in excess of the correct value. In the previous example the agreement was almost perfect (1002 compared with 1000), and these two comparisons give some indication of the order of accuracy obtained when two graphical steps are involved, viz., that of obtaining z from semi-log plot of Eq. 7(3) and that of estimating the distance from the vector mean centre to the typical mid-point P, (See Fig. 7(4).

In method (ii) the final result is obtained from the estimate of "displacement" due to winds within a particular cell - in this case, however, a constraint is placed on the total "displacement" in the course of the estimate, and thus the agreement between the 894.2 and the correct value 896 is to be expected.

The radii of the frequency density contours corresponding to z = 0.025, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 were also obtained and those for z = 0.8, 0.6, 0.4, 0.2 and 0.1 are drawn in Fig. 7(7).

We shall now consider some estimates of total displacement within various speed categories (i.e. for "all directions") for the two cases already discussed.

Table 7(7). Estimate of total displacement (nautical miles) within certain speed categories as obtained by several methods, for the distribution of Example 1 defined in Table 7(5), derived:

(i) from "GM 85" (p. 6, Table IV);
(ii) from the frequency density distribution;
(R) from frequencies given by the "Rand" Tables (using $(\sigma/|\bar{\mathbf{V}}|)$.

Method	0-3	3-14	14-27	27-41	41-54	54-68	68-81	81-95	95	Total (a)
	Nautical miles.									
(i)	23	957	5654	9894	8673	5344	2041	616	(100)	33302
(ii)	10	1058	5590	10183	8431	5380	1945	616	..	33213
(R)	12	995	5658	10030	8502	5490	1937	704	(100)	33428

(a) The theoretical value, $N|\bar{\mathbf{V}}|/q$ = 33,000 nautical miles.

Turning to the "Bell Rock" series, we have the values of Table 7(8).

Table 7(8). Estimate of total displacement (nautical miles) within various speed categories for the distribution of Example 2 defined in Table 7(6), as obtained:

(i) by the use of the frequency density distribution;
(ii) by direct summation of displacements within appropriate speed categories;
[R] from the frequencies computed from the "Rand" Tables assuming the value $[\sigma/|\bar{V}|]$
(R) as for [R] except that the value $(\sigma/|\bar{V}|)$ used.

Method	$\frac{1}{2}$–$6\frac{1}{2}$	$6\frac{1}{2}$–$12\frac{1}{2}$	$12\frac{1}{2}$–$18\frac{1}{2}$	$18\frac{1}{2}$–$24\frac{1}{2}$	$24\frac{1}{2}$–$30\frac{1}{2}$	$30\frac{1}{2}$–$36\frac{1}{2}$	$36\frac{1}{2}$–$42\frac{1}{2}$	$42\frac{1}{2}$–$48\frac{1}{2}$	Total (a)
	Speed categories (knots) — Nautical miles.								
(i)	333	1972	3655	3943	2989	1782	750	227	15651
(ii)	384	1917	3514	3837	3070	1771	849	278	15620
[R]	326	1907	3527	3911	3031	1802	814	375(b)	15693
(R)	342	1898	3526	3909	3030	1742	770	328(b)	15545

(a) The theoretical value is 15717 nautical miles.
(b) Includes contribution for higher speed categories.

Thus in Example 1 the various methods result in an over-estimate of about one percent, and in Example 2 all methods give a slight under-estimate of about one percent or less.

From the material presented in this section, it is considered that a method based on the frequency density surface, or one upon a purely graphical technique, gives results in sufficiently good agreement to justify using either as convenient when constructing two-dimensional frequency and "flow" fields for a normal circular distribution of stated σ, $\bar{V}$ and N. The contrast between estimates [R] and (R) indicate the cumulative effect of adopting the slightly different alternative values for the expression $\sigma/|\bar{V}|$.

Section 5. Methods for the diagrammatic analysis of actual wind distributions: comparison with equivalent normal circular distribution.

We shall now describe the main graphical techniques used to analyse certain observational series. To illustrate these, the data for "Bell Rock" already employed in the previous section, provides the material for the discussion.

Three types of diagrammatic representation were utilised and are illustrated in Figs. 7(6), 7(7) and 7(8). For this particular series the actual value of $\sigma/|\bar{\mathbf{V}}|$ was used when required, and not the slightly different value $\left[\sigma/|\bar{\mathbf{V}}|\right]$ (see p.109).

In Fig. 7(6) are plotted:

The frequencies and displacements within each of the 12 directional categories, together with the values appropriate to a NCD of given N, q and ($\sigma/|\bar{\mathbf{V}}|$):

The mean speeds for each of the 12 sectors, and a circle (to which the term "characteristic circle" is now given), which intercepts the radii from the origin O at points whose distance from O equals the mean speeds in those directions, for the appropriate NCD.

In Fig. 7(7) three sets of isopleths are plotted:

A series of irregular, and generally closed, curves (viz. the lightly pecked lines) giving the frequency density contours of the actual distribution (numerical values are entered for the highest wind speed category in each of the basic directions):

A set of circles, centred on the vector mean, and giving selected frequency density contours (viz. the continuous lines) for the equivalent NCD:

A series of isopleths (i.e. the heavily dashed lines) obtained from the above two sets by graphical substraction, to delineate well-marked regions of positive or negative anomaly of the actual from the theoretical (viz. the NCD) pattern. In diagrams of this type, anomaly isopleths were not drawn when it appeared that a slight shift in the position of the two basic sets of lines would result in non-intersection of frequency isopleths.

In Fig. 7(8) there are also three sets of isopleths:
A series of irregular, and generally closed, curves (viz. the lightly pecked lines), giving the pattern of displacement based on the actual data:
A more regular series of isopleths (viz. the continuous lines), disposed about an elongated, and often crescent-shaped, nucleus, which corresponds to the flow pattern of the appropriate NCD:
A series obtained (viz. the heavily dashed set) by graphical substraction, to delineate regions of positive or negative anomaly.

All three types of diagram may be constructed either preserving the original absolute magnitudes or on a relative (per mille) basis: absolute values have been retained in the particular cases now to be described.

In Fig. 7(6) overleaf, totals of frequency and displacement suggest relative maxima in the sectors centred around:

$270^{\circ} - 300^{\circ}$; $30^{\circ} - 60^{\circ}$; 180°

with associated deficits in the sectors about

360°; 120°; $210^{\circ} - 240^{\circ}$.

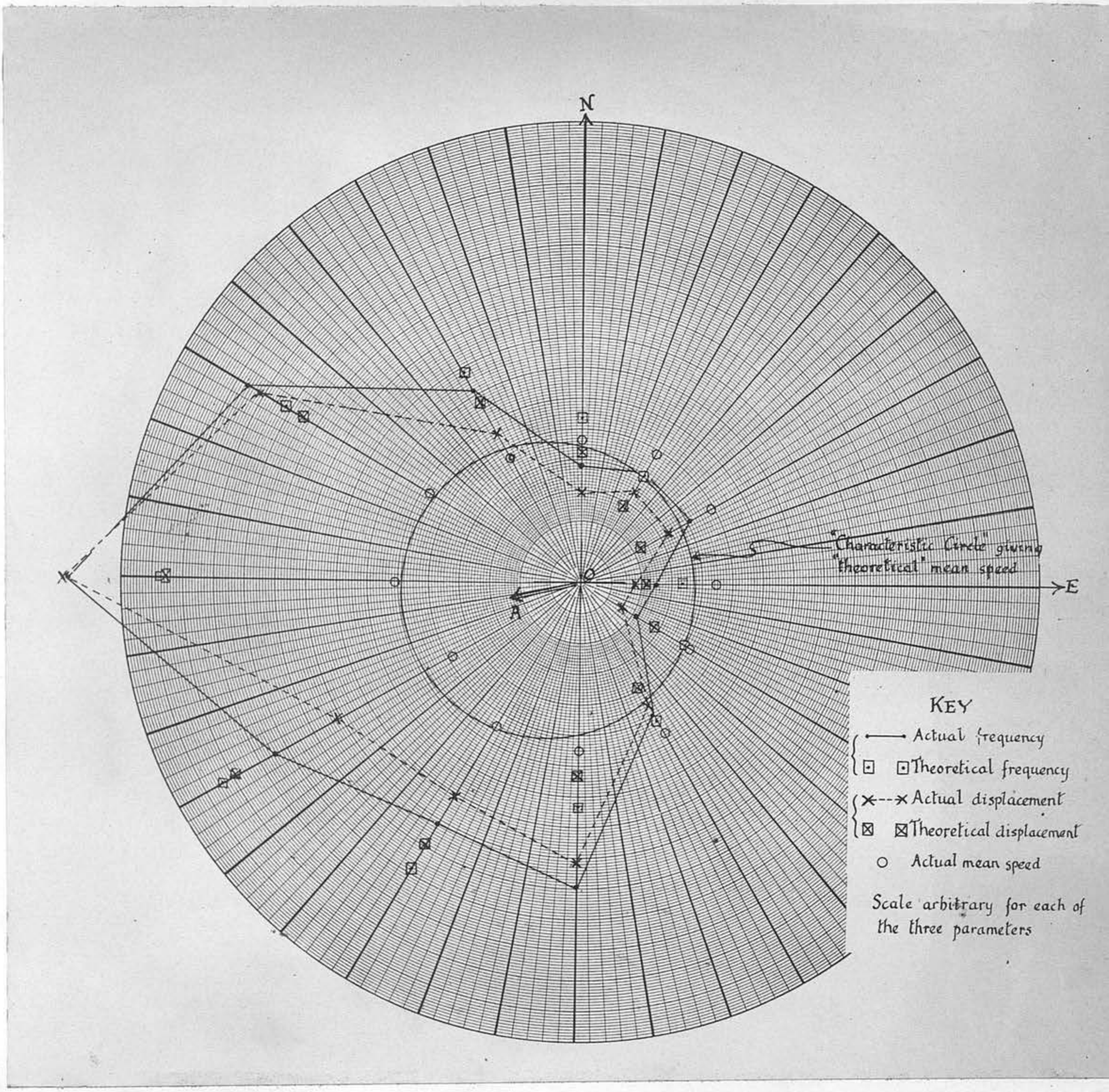


Figure 7(6). Polar diagram showing distribution by direction, for 30^{o} sectors; of frequency, displacement and mean speed (Bell Rock, September, October, November, 1951-55)
"Theoretical" values - those for the normal circular distribution for which the parameter q is obtained from the data .

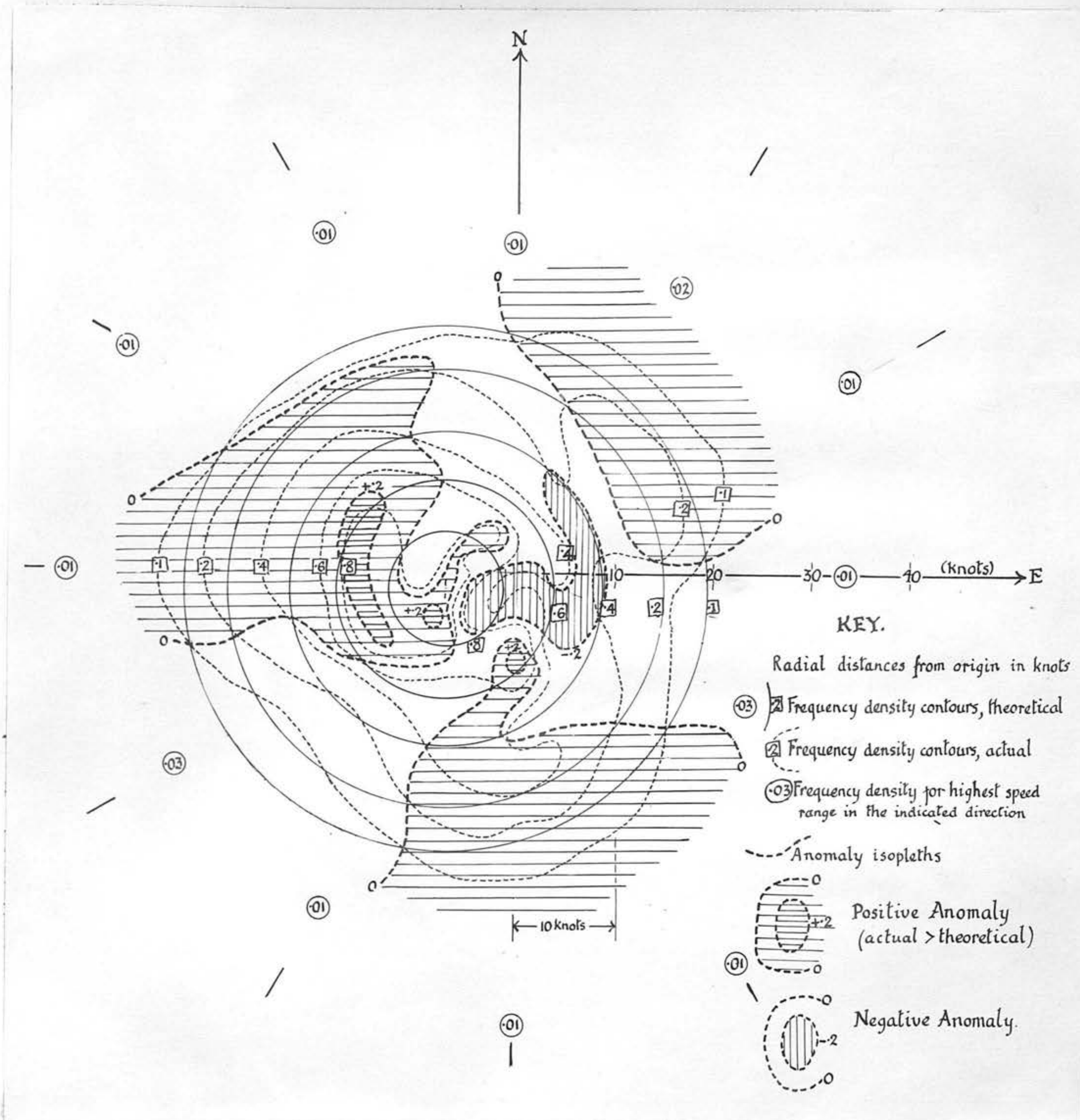


Figure 7(7). Frequency density diagram showing actual and "theoretical" density isopleths and anomaly pattern: together with frequency density in the highest speed category in the several directions.
(Bell Rock, Sept. - Oct. - Nov. 1951-55).

There is evidence that mean speeds are comparatively high in the 30°, 60°, 150°; 90° and 120° sectors, although for the last two mentioned sectors the displacement is small both absolutely, and relative to the values associated with the corresponding NCD. It would appear that winds from an easterly point, although few in comparison with those from a westerly point, nevertheless include some in the higher speed ranges (or alternatively lack a due proportion of winds of moderate speed or less).

Frequency density is examined in Fig. 7(7), (p. 116). The lightly pecked line sketches the actual density contours for $\bar{z}$ = 0.8, 0.6, 0.4, 0.2, 0.1. Although there is considerable irregularity in the actual distribution, isopleths are readily drawn to reveal a coherent picture showing the alternation of sectors of maximum and minimum frequencies. Comparison with the theoretical density contours, viz. the circles, is aided by isopleths of anomalies, from which the concentration of winds of moderate strength in the 270° and 180° sectors is evident, as is the scarcity of light winds from the eastern sectors - a conclusion in conformity with that already deduced from mean wind speeds.

Although a considerable subjective element arises when drawing isopleths, there can be little doubt concerning the existence of a positive anomaly, (i.e. actual frequency > theoretical frequency) from 240° to 320°, particularly at moderate speeds (12 - 18 knots), and a negative anomaly from about 20° to 140° at speeds of less than 12 knots. The positive anomalies for the north-east sector at speeds greater than 12 knots and for the southern sector are also reasonably well established; the negative anomalies in the south-east, north, and, particularly in the south-west, sectors are not so well established, although in order to compensate for the definite excess elsewhere, it is almost certain that such deficits are real.

It is worth noting that to embrace the highest value in the directions 60°, 150° and 180°, a circle of radius 2.67σ needs to be drawn, whilst to embrace the extreme values at 270°, 300° and 330°, one of 2.26σ is required; (with an NCD the probabilities of values outside a radius of 2.26σ and 2.67σ are respectively less than seven in one thousand and less than one in one thousand). The highest values are thus by no means confined to directions near that of the vector mean, nor to the most frequent direction.

The distribution of "flow" or "displacement" is indicated in Fig. 7(8) shown overleaf. Isopleths have been sketched (the lightly pecked lines), but in these diagrams the area enclosed within an isopleth has not the significance as is attached to area in a frequency density diagram. If the number of directional categories is changed, and/or the speed intervals altered, the isopleths would be distorted, although the general position of the main centres and the basic shape of the pattern remain. If the size of the directional intervals is unchanged, but the axes rotated, then the interceptions of the isopleths and the new directional axes will give the appropriate values for $\mathcal{D}$.

Patterns similar to those in the frequency density diagram are revealed - certain contrasts, however, being exaggerated, the tendency being for areas of maximum concentration to be shifted towards higher speeds.

The elongated nucleus, and the isolated area of low values round the origin, are typical.

of what?

From this diagram positive anomalies are again evident, located in the directions 10° to 70°; 150° to 180°; 260° to 330°. In the second

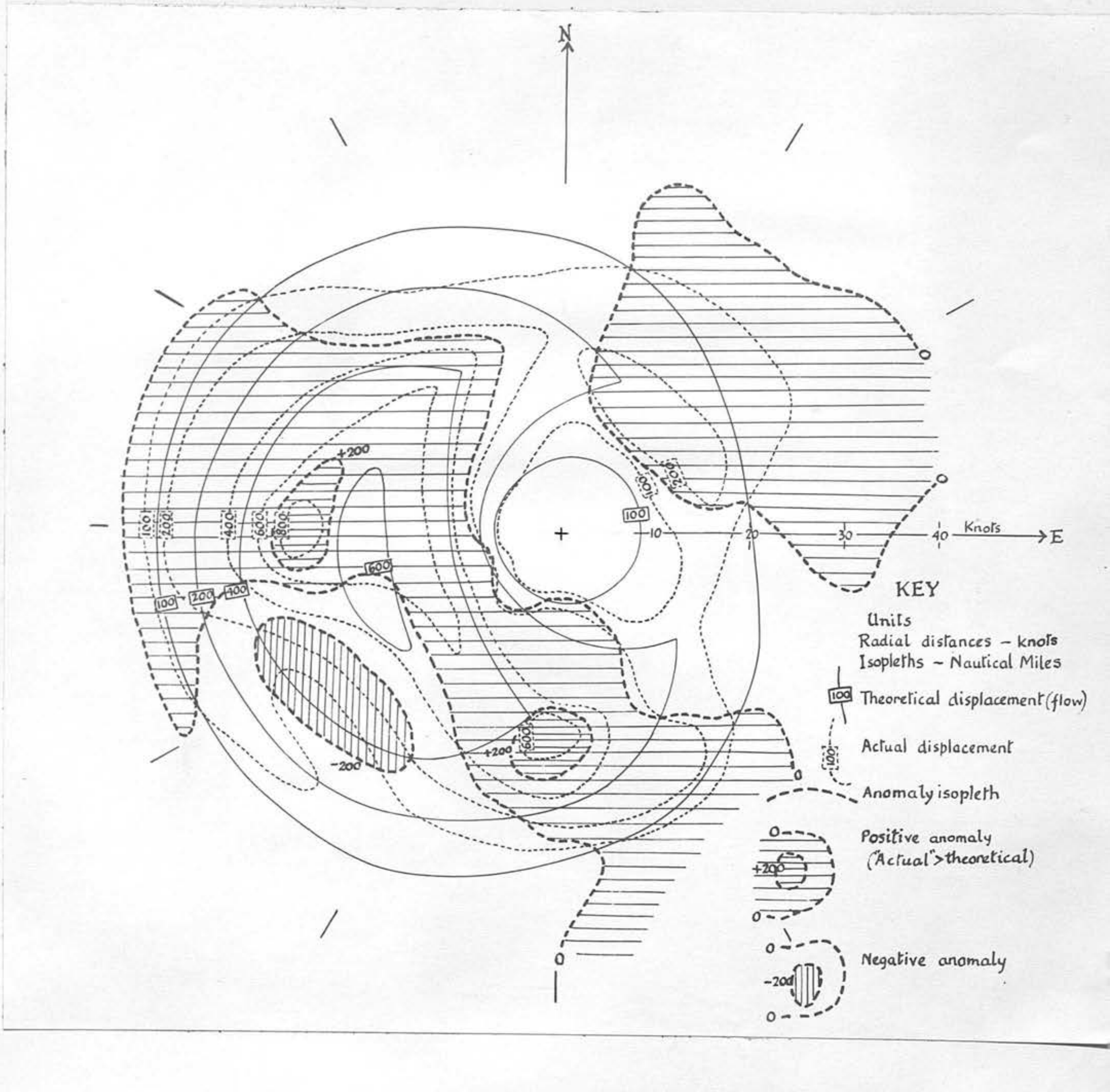


Figure 7(8). To show actual and "theoretical" isopleths of "flow" or "displacement" (based on 12 sectors each of 30^{o}, and speed intervals of 6 knots): together with anomaly pattern. (Bell Rock, Sept. - Oct. - Nov. 1951-55).

two instances the excess is largely due to separate concentrations of winds of moderate strength, whilst the excess for the 10° to 70° sector seems to be due to a relatively high frequency of winds of moderate or greater strength. The deficiency of "flow" from the south-west is rather more evident than in Fig. 7(7), and that in the 360° to 100° sectors less evident, in this method of representation. Again, however, the positive anomalies in the southern and western sectors are accentuated by an excess of winds of moderate strength.

Chapter 8

Some Details concerning the Properties and Treatment of the Observational Material.

It is now desirable to discuss certain details relating to the observational series used in Chapter 9 to examine the potentialities of the methods of analysis already described. Clearly in the attempt to establish, or at least to identify, those properties of the two dimensional wind field which might be ascribed to synoptic and to topographical factors, it is necessary that the observational stations should be as free as possible from purely local obstructions, and capable of producing high quality observations throughout the 24 hours. The selected stations are listed in p. 65, and further details regarding the several sites are set out later in this chapter.

It was also essential to limit the work of extraction and computation, and various procedures were adopted for this purpose: the accuracy and adequacy of the resulting samples are discussed in Appendix II(c).

As regards the Ocean Weather Ship "Juliet" series, wind observations for all eight synoptic hours combined, and separately for 0300GMT and 1500GMT, were extracted mechanically according to Beaufort force and a 32-point compass by the Marine Branch of the Meteorological Office - the month of given name being the basic period. For the shorter North Greenland series at "Northice" attention was concentrated on the observations at 0001GMT (or in their absence at 2100GMT), 0600, 1200 and 1800GMT, (N.B., "Northice" is not an official designation, but has been used as a convenient term in published accounts of the British North Greenland Expedition 1952-54; e.g. Hamilton and Rollitt, 1957; Hamilton 1958(a)). It might also be noted that the observations attributed to a particular hour were made 30 minutes before the hour.

For both the Bell Rock and Lerwick series, the observations of mean hourly wind speed and direction for the 60 minutes ending 0400GMT and 1600GMT were extracted. These hours may not be the most suitable two hours for identifying, and by combining for eliminating, a diurnal variation, but, given the need to select, and the probable future need to link surface wind distributions with upper winds at moderate heights obtained from daily soundings commencing 0300 and 1500GMT, it was considered that the best choice had been made. In passing it may be observed that for purposes of correlating surface winds with those revealed by balloons during the first few thousand feet of ascent, "spot" observations at 0300 and 1500GMT are more appropriate than those at the succeeding hours, whilst, given mean hourly wind speeds, the 60 minutes ending 0400 and 1600GMT are at least as appropriate as the preceding 60 minute periods.

The results presented in Appendices II(c) and II(d) are held to demonstrate that the sampling techniques adopted proved to be very satisfactory for present purposes.

As to the number of years record needed for a satisfactory sample, Brooks et al (1950) state that, in respect of upper winds, "a fair number of observations spread over a period of 10 years or more gave a better indication of the normal value than a larger number of daily observations for a period of only 2 or 3 years". In the present work the marine winds were for two groups of three years, viz., 1950-52 and 1953-55, those for Bell Rock and Lerwick for five years (respectively December 1950-November 1955, and January 1951-December 1955), whilst those for "Northice" were necessarily confined to the period of the Expedition (November 1952-June 1954).

Section 1. Ocean Weather Ship "Juliet" Series (mean position, $52\frac{1}{2}^{\circ}$ N. 20° W.; period, 1950-1955).

(a) Position, observational techniques, and the reduction of data.

The positions taken by the several ships involved in maintaining station "Juliet" lie within an area defined by (50° - 55° N., 15° - 25° W.), the normal position being the central point ($52\frac{1}{2}^{\circ}$ N., 20° W.), from which the nearest land is about 400 miles to the east-south-east, and, the closest land, about 200 miles from the Irish coast. Enquiries have confirmed, however, that rarely are reports received from the ship when off the central point.

In contrast to the data for Bell Rock and Lerwick, which relate to the mean wind speed and direction estimated from a 60 minute record on an autographic chart, those from the Weather Ships derive from visual estimates of the mean position of an oscillating needle taken over a period of at least 15 seconds, and corrected for the speed and course of the ship. In addition, however, before the instrumental reading (in knots and degrees) is accepted, an estimate is made, in consultation with the Officer-on-Watch, of the wind speed (on the Beaufort scale) judged from the state of the sea "account being taken of the lag between the wind getting up and the sea increasing", (Shellard, H.C., 1958 - Private Communication, from which these notes on observational technique at sea are derived). The mean speed and direction so obtained is then expressed in terms of Beaufort force and 32 directional points. Hence, although some of the precision of the original record is absent from the material now available for discussion, the successive steps in its reduction are such as to eliminate much short period variability, leading to a result probably fairly comparable with the mean hourly wind derived from the analysis of an autographic chart.

The height of the anemometer on the several ships varied, but it appears to have been between about 50ft. and 65ft. above the level of the sea. Even were it judged desirable, information sufficient to effect a correction to some given height was not available, and would in any case have entailed an excessive amount of additional computation for a gain in accuracy which, in view of other shortcomings in the techniques of observation and reduction, could only be considered of doubtful value.

In view of the discussion in the previous paragraphs, objection might be raised to the number of figures retained when expressing results, e.g., as in Appendix II(d); however, by so doing, it is often possible to indicate variation arising solely from causes such as the choice of hour(s) of observation and the accuracy of the various numerical and graphical procedures. It is also suggested, in respect of the non-dimensional quantities q, $\sigma/|\bar{V}|$, $s/\overline{V}(s)$, that more knowledge of the properties of these expressions is needed before a confident decision can be made as to the number of significant figures which are appropriate, and the number having physical significance. In arriving at the results set out in Appendix II(d), the original directional classification into 32 "points" was retained and it was not considered necessary to use the grouping "correction" for direction mentioned in Appendix II(b), p. 3. For the final graphical analysis on a seasonal basis, the data were aggregated into the standard twelve 30 degree sectors.

(b) Some features of the numerical results.

A list of values of the more important parameters appears as Section 1 of Appendix II(d).

Values for each month of a given name for "all hours", for 0300 and for 1500GMT, according to the conventional "seasons", viz., "spring", March - May; "summer", June - August; "autumn", September - November; "winter", December - February; are given for each of the three year periods and finally for the complete six-year period. Some results relevant to the present enquiry will now be noted.

(i) Comparison between results for 0300GMT and 1500GMT with those for "all hours" for month of given name.

For the parameters $\overline{V}(s)$ and $\sigma(o)$ there is an approach to two-figure agreement: such a level of agreement is also often achieved in respect of s, $\sigma(N)$ and $\sigma(E)$.

The numerical differences between the various estimates for $|\overline{\mathbf{V}}|$ rarely exceed about two knots, but is just as great at low values (e.g., <five knots) as at the higher ranges.

Estimates of ϕ (the direction of the vector mean) rarely differ by more than 10^{o}. Values of q for a month of given name generally lie within a range of 10%: the largest discrepancy being for November 1953-55 with estimates of:

37.2% (0300GMT); 50.4% (1500GMT); 41.6% ("all hours").

Differences between estimates of $\sigma/|\overline{\mathbf{V}}|$ may eventually be judged significant, but only rarely do the three given estimates lie outside the tolerance limits suggested by Brooks et al (1950).

(ii) Period comparisons of monthly and seasonal values ("all hours") series.

A formal analysis of variance has not been attempted, but it seems clear that the "between-period" variability is large compared with that discussed in sub-section (i) above, and certainly no less than the "between-month" variability.

The "between-period" variability appears to be larger in "spring" than in the other seasons, with April the most stable of the three months in question. The relatively low value of q for the "spring" months reflects this variability.

Section 2. "Northice" series. ($78^\circ 04'$ N., $38^\circ 29'$ W., height above m.s.l. 2343 m, period, November 1952 to July 1954).

(a) Site, observational techniques and the reduction of data.

The station "Northice" was maintained by the British North Greenland Expedition during the 1952-54 Expedition, observations of surface wind being obtained as far as possible for each of the eight synoptic hours during the period 1 November, 1952 to 15 July, 1954, although only observations for three of the hours, viz. 0600, 1200 and 1800GMT ran unbroken throughout the period.

At the site "the inland ice is completely featureless and the slope indiscernable by eye. The maximum slope of the surface measured at a point near "Northice" was found to be $+9.1'$, or about 2.7×10^{-3} upwards toward true direction 240°. There is no ice-free land within 300 km of the station". (Hamilton et al, 1957). By usual standards, therefore, the site is quite free from obstructions.

From 1 November, 1952 to 18 April, 1953, the wind speed was measured by a cup-generator anemometer two metres above the surface, and after that date at the standard height of 10 metres: the earlier values were increased by 30% to allow for the height difference before publication. Wind velocity was recorded in knots and in tens of degrees.

For the present investigation it was eventually decided to rely on a sample obtained by pooling two groups of observations, one comprising those for 0001 (or failing that 2100) and 0600GMT, and the second those for 1200GMT and 1800GMT. The adequacy of this sample is considered in Appendix II(c), Section 2, and reasons given to justify regarding it as a representative one.

The class intervals for speed were:

$< \frac{1}{2}$; $\frac{1}{2} - 3\frac{1}{2}$; $3\frac{1}{2} - 6\frac{1}{2}$ etc. in steps of 3 knots

and 30 degrees for direction. The "grouping" correction for direction (see Appendix II(b)), was employed.

(b) Some features of the numerical results.

The most striking feature of the seasonal results detailed in Appendix II(d) Section 2, is the high values of q viz:

"spring" 93.8%: "summer" 76.4%: "autumn" 89.9%: "winter 85.5%

in sharp contrast to those for other land stations.

The mean direction is particularly steady throughout the year compared with results for the other series.

Section 3. Bell Rock Series. (56°26′N., 2°24′W., period December 1950 to November 1955.

(a) Site, observational procedure and reduction of observations.

The general topographical setting is illustrated in the sketch map which forms Fig. 8(1) overleaf. Within the sector 020° - 090° - 155° there is an uninterrupted fetch of about 300 miles over the open sea; thus there is a complete absence of obstruction for winds having directions included within three of the standard ranges centred at 60°, 90°, 120°, and practically complete absence for the 30° category. The nature of the fetch for the other categories may be gleaned from the figure.

The anemometer head stands at 130ft. above sea level and, as Goldie (1935) has shown, the air stream at this height is almost certainly free from interference from the lighthouse structure. It is not customary to attempt to reduce these observations to the nominal 30ft. height of most land-based instruments, and no such procedure is adopted here.

Owing to damage to the instrument in December 1955, data for December 1950 have been substituted to complete a 60 month sequence.

In the original tabulations speeds are quoted in knots, and the class intervals used for the analysis were:

$<\frac{1}{2}$; $\frac{1}{2} - 3\frac{1}{2}$; $3\frac{1}{2} - 6\frac{1}{2}$, and thence by six knot intervals. It is stated on the original tabulations that, for reported speeds of three knots or less, the indicated direction may be unreliable; in order, however, to keep to a minimum the number of statistically troublesome entries of speeds without direction, the indicated direction was accepted.

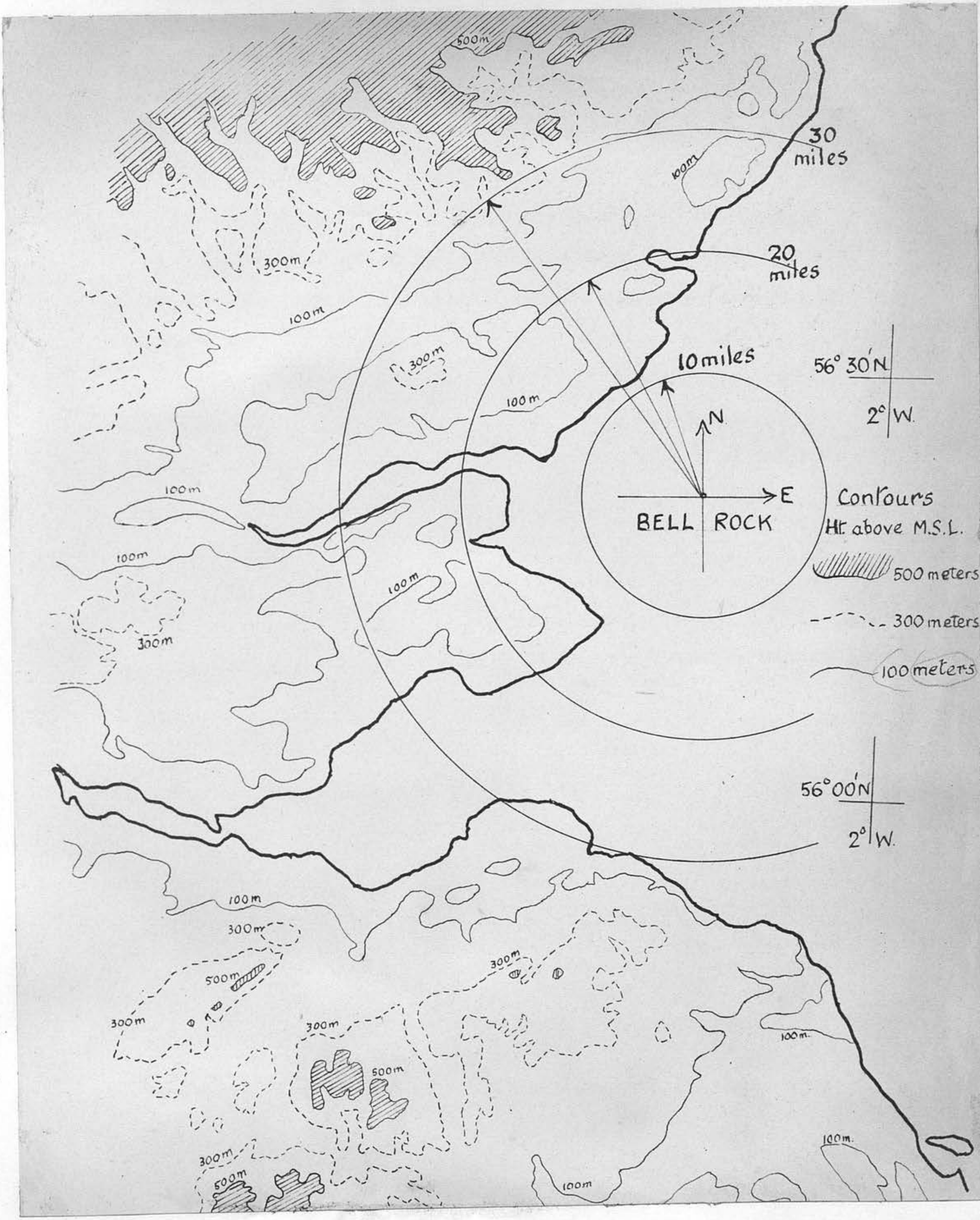


Figure 8(1). To show the general setting of Bell Rock Lighthouse.

In those cases in which no direction was given (appreciably less than half the total of winds of three knots or less), a direction was selected by reference to the general run of the figures. The directional grouping correction of Appendix II(b)(p. 3) was adopted.

Mean daily speeds and period "run-of-wind" were also extracted for purposes of examining the reliability of the chosen sample (see Appendix II(c), Section 3).

(b) Some features of the numerical results.

From the tabulated result in Appendices II(c) and II(d) certain properties of the series are evident:

(i) Mean speed for a sample of five months of a given name obtained from data grouped into six knot categories, differs negligibly from that computed from ungrouped data.

(ii) Mean speeds for five or more months of given name obtained from the average of the 0400 and 1600GMT observations, differ by only a few tenths of a knot from the true mean speed obtained by observations for all 24 hours.

(iii) Taken in conjunction with Goldie's (1935) results mentioned in Appendix II(c)3, it seems probable that for samples of five or more "autumns" or "winters", the mean direction is given to within a few degrees by the mean direction for either the 0400 or 1600GMT series, and it may also be surmised that such a result might be generalised to refer to any hour.

In "spring" and "summer, however, there is a persistent diurnal effect, marked in the case of direction, and present in respect of speed:

the diurnal variation is, it would appear, reasonably well eliminated by pooling data for the two chosen hours. Accordingly it is reasonable to suggest that the mean of the 0400 and 1600GMT readings give a tolerably good approximation to true seasonal means, providing at least a five years record is utilised.

(iv) Either of the two sub-series will give s to one significant figure and $\sigma_{(O)}, \sigma_{(N)}, \sigma_{(E)}$ approximately to two figures.

(v) It would seem that, on a seasonal basis, a value of q exceeding 50% is probably unusual for any but exceptionally sited land-based stations. It is worthy of note that in this, as well as in the OWS "J", and the Lerwick series, the lowest value of q occurs in "spring"; furthermore the numerical values are not dissimilar, viz. 10.5%; 16.7% and 15.1% respectively.

(vi) For each "season" (for the sub-series as well as the combined series) the point defined by $q : \sigma/|\overline{V}|$ lies well within the "Brooks" tolerance.

(vii) $s/|\overline{V}_{(s)}|$ is high relative to the theoretical (i.e. the NCD) value, especially in the "spring" and "summer" (it runs at an even higher level for Lerwick) and this may be characteristic of land stations, where we expect a high percentage of calms, which contribute nothing to the total displacement and so reduce $|\overline{V}_{(s)}|$. Other possibilities will be mentioned later, but this is just the type of "characteristic" numerical property which, it is suggested, should be investigated.

Section 4. Lerwick Observatory Series (60° 8′ N., 1° 11′ W. height above m.s.l. 269 ft., period 1951 - 1955).

(a) The site, observational techniques and reduction of data.

The general topographical setting of the station is indicated in the sketch map in Fig. 8(2). South of the Observatory, the Shetland mainland is confined to a narrow peninsula less than about six miles from east to west and extending southwards for about twenty miles. A longitudinal range of hills of general height 500 ft. with a number of summits to 750 ft. or more, forms the backbone of the peninsula and is encountered a few miles west of the Observatory. Northwards the land mass broadens rapidly to some 20 miles and continues thus for about thirty miles - although much broken by inlets from the sea - eventually terminating in a large number of small islands. Much of the region north of the station reaches more than 300 ft. above m.s.l.

A relatively unobstructed fetch over the open sea exists within the sector 125° - 180°, and the open sea is also reached towards the north-east after a few miles of land track which falls away from station level.

Winds approaching the station from an easterly point are presented with a steep rising gradient from sea level to 250 ft. which terminates about a quarter mile from the anemometer. "..... there is a downward slope for about a quarter mile extending to the Lock of Trebister on the south-west, Sandy Loch to the north-west, and to the Burn of Sound to north-north-west; beyond these and distant about three-quarters of a mile from the Observatory are small hills, Munger Hill to the south is

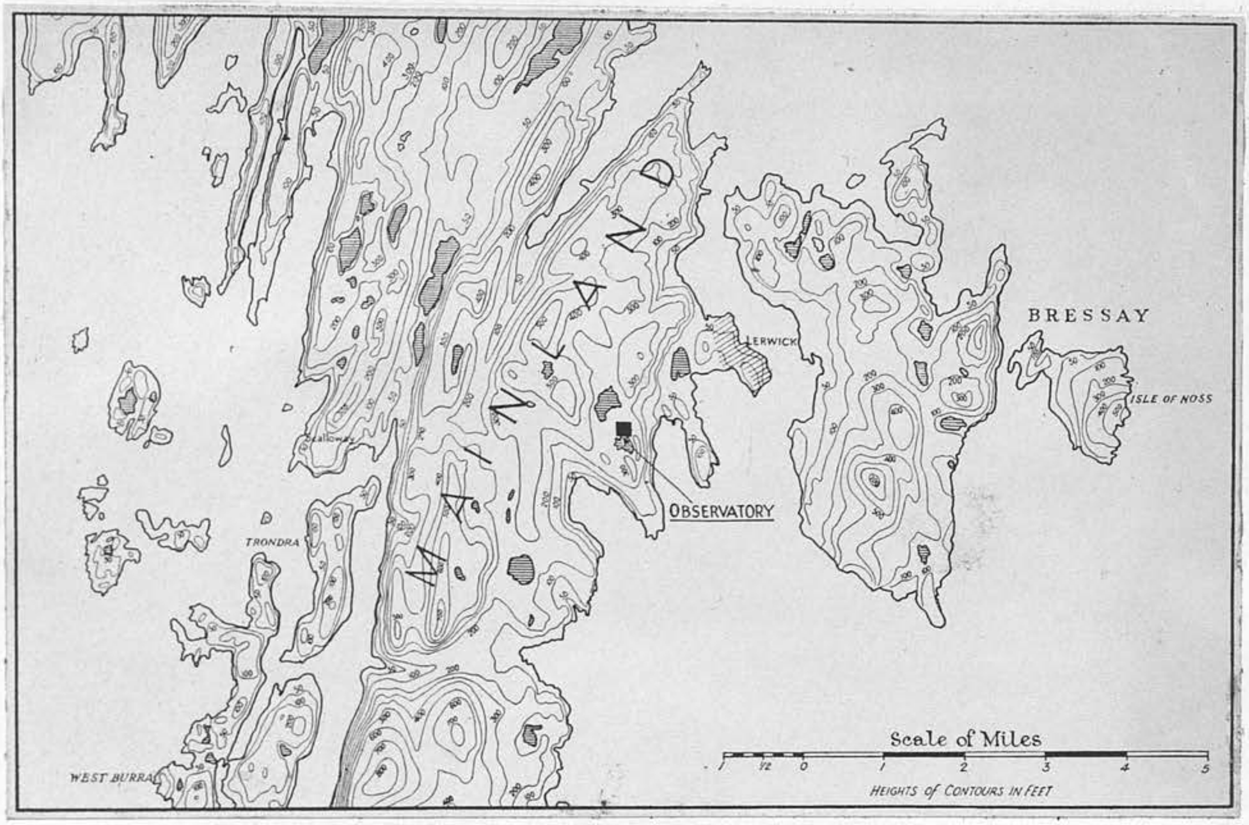


Figure 8(2). To show the general setting of Lerwick Observatory.

(Reproduced from the Observatories Year Book 1935 by permission of the Controller, Her Majesty's Stationery Office, London).

about 320 ft. above m.s.l., Shurton Hill to the west-north-west rises to 576 ft., and Stoney Hill to the north to about 400 ft.". (Observatories Year Book, H.M.S.O., 1935).

The anemometer head is 53 ft. above ground level with an "effective height" of 37 ft.

During the period under discussion, observations were tabulated in tens of degrees and in metres and tenths per second. For speed, the lcass intervals were:

metres per second.

< 0.3 ; $0.3 - 1.55$; $> 1.55 - 3.05$; $> 3.05 - 6.0$,

and thence by steps of 3 m/s. The relatively few instances in the speed class 0.3 - 1.55 m/s for which no direction was specified, were dealt with as were similar cases for Bell Rock, (p. 128). Monthly run-of-wind was also extracted to provide column (iv) of Table II(c)(4).

(b) Some features of the numerical results.

(i) Monthly speeds at 0400GMT are less than those reported at 1600GMT, the differences being proportionally and absolutely greater in the six months April to September than in the other months.

(ii) The above contrast obviously reappears on a seasonal basis but, unlike Bell Rock, the directional shift is very small although in both "spring" and "summer" the mid-afternoon direction is backed relative to that for the 0400GMT observations.

These facts prompt a number of questions.

Are the Shetland Islands sufficiently far from the main mass of the British Isles to be regarded as climatically "isolated"?. (This result might not be obtained were the Shetlands situated other than to the north of the mainland).

Is the land mass of the islands insufficient to generate specific diurnal circulations?-although it should be noted that the small degree of backing is consistent with a day-time inflow towards the bulk of the land mass superimposed upon a mean westerly drift; as is the case at Bell Rock. (The general windiness of the region might, of course, mask the necessarily rather restricted specific effects).

Or have we here an instance in which a number of influences compensate each other?

(iii) In "autumn" and "winter", agreement between the two sub-series suggest that, on a seasonal basis, the mean of a single daily observation will provide an estimate of the parameters $\overline{V(s)}$, $|\overline{\mathbf{V}}|$, s, $\sigma_{(o)}$ to an accuracy generally better than that of one significant figure, and of ϕ to the nearest ten degrees.

At this stage of our knowledge, it does not seem profitable to offer any firm suggestions regarding q, $\sigma/|\overline{\mathbf{V}}|$ and $s/\overline{V(s)}$.

(iv) From the internal evidence implicit in the tables in Section 4 of Appendices II(c) and II(d), and the conclusions relating to Bell Rock, there seems no evidence to reject the assumption that the mean of the 0400 and 1600GMT observations provide an adequate sample of the wind distribution.

Chapter 9.

Results of a Statistical and Graphical Analysis of Four Series of Surface Wind Observations

We shall now discuss some results of applying, to the observational series described in Chapter 8, the graphical and statistical techniques mentioned and/or developed in the previous chapters.

For each season, and for each of the four main series, a polar graph of the type illustrated in Fig. 7(6) was prepared; for all cases a frequency density diagram, Fig. 7(7); and for a number of cases the flow, or displacement, diagram of Fig. 7(8).

Further attention will also be given to the properties of the three non-dimensional quantities.

Section 1. Some preliminary considerations.

It is provisionally assumed that the normal circular distribution will prove a useful theoretical model against which to compare the actual distributions. For this theoretical distribution there is a unique relationship between the parameters

$$q \quad , \quad \sigma/|\overline{\mathbf{V}}| \quad , \text{ and } \quad s/\overline{V(s)} \quad ,$$

and the question has already arisen as to which of these quantities shall be regarded as the independent variable, when developing the frequency density and the displacement diagrams etc., of the equivalent NCD.

The value of q is more immediately obtainable from a wind tabulation than is σ , but it may well be argued that, nevertheless, σ is the more characteristic parameter. The decision as to whether to adopt the actual value of $\sigma/|\overline{\mathbf{V}}|$ which we are writing $(\sigma/|\overline{\mathbf{V}}|)$ or the theoretical value, i.e., $[\sigma/|\overline{\mathbf{V}}|]$, corresponding to the computed q , has extensive implications: the quantity σ occurring in R (the radius of the "characteristic" circle); in the computation of the frequency density and, by one method, the displacement, fields; and in the Knighting and "Rand" checks on marginal frequencies.

We are, of course, at liberty to adjust the value of $|\overline{V}|$ so that

$$\left[\sigma/|\overline{V}|\right] \equiv \left(\sigma/|\overline{V}|\right)$$

but it seems that at this stage in the development of our ideas, the vector mean has a strong claim to be regarded as a fundamental characteristic of any distribution, and in the present work it has been decided to regard $\overline{V}(s)$ $\overline{V}$ and hence q, as the basic variables of the distribution, implying for the NCD, values for $\left[\sigma/|\overline{V}|\right]$ and $[\sigma]$ differing more or less from the actual values.

One of the more important operations, the construction of the frequency density circles, is fortunately relatively insensitive to variations in σ for, as is evident from the expression,

$$z = \frac{N}{\pi\sigma^2} \exp(-v^2/\sigma^2)$$

there can be a measure of compensation.

In general, the distribution of frequencies and displacements according to direction and speed, is expressed in terms of parts per thousand, a procedure which obscures the few percent differences between the actual and theoretical values for these quantities - the values for the NCD being typically built up from basic parameters and hence subject to a series of errors of estimation etc., (for the magnitude for some of these errors see Tables 7(6), 7(7) and 7(8)).

For brevity we shall use the terms "theoretical" or "expected" value, when referring to the value of any quantity (e.g. frequency, displacement, or mean speed), appropriate to a normal circular distribution of parameter q equal to that of the observed series: "excess" and "deficit" are also with respect to the NCD unless otherwise indicated.

Initial experience of constructing frequency density diagrams revealed a tendency for a high concentration in the lowest speed category; in order to "smooth out" irregularity, entries in the lowest two speed categories were pooled and attributed to the mid-value of this larger speed range (it will be recalled that the two lowest speed classes used for tabulation were usually formed by sub-dividing the basic interval for the series, see Chapter 8). Entries of "calms" also lead to a similar difficulty, and the "peak" concentration in the immediate vicinity of the origin has been ignored when drawing isopleths. Entries in these lowest speed ranges do, of course, influence the value of the computed parameters, and due weight is accorded to them in respect of numerical results.

The mapping of displacement does not encounter these difficulties, since the contribution of "calms" is necessarily zero and that of the light winds relatively small.

<u>Section 2.</u> <u>Ocean Weather Ship series.</u>

(a) The distribution of frequencies (F) and displacement (D) by direction.

The data now to be examined are set out in Table 9(1) overleaf, and the following points may be noted:

In "summer", "autumn" and "winter" both (F) and (D) have a single maximum and a single minimum, and the extreme values of both parameters occur together in the same sector, viz:

"autumn" and "winter"	maximum in the 270° sector minimum in the 090° sector
"summer"	maximum in the 300° sector minimum in the 090° sector

Table 9(1). Frequencies (F) of surface wind observations, and of displacement (D); by direction in intervals of 30°, centred as indicated. Ocean Weather Ship "Juliet", 1950-55:
(i) observed values;
(ii) values for a normal circular distribution, of parameter q derived from the actual series.

Central Direction (degrees).		Spring.		Summer.		Autumn.		Winter.	
		F	D	F	D	F	D	F	D
		(In parts per thousand).							
360	(i)	64	65	56	52	49	43	43	37
	(ii)	-	64	54	48	-	52	-	49
030	(i)	60	55	31	25	38	30	32	27
	(ii)	-	60	37	29	-	33	-	34
060	(i)	55	50	21	14	33	26	27	27
	(ii)	-	58	30	22	-	26	-	25
090	(i)	53	49	20	14	30	24	27	26
	(ii)	-	63	30	20	-	25	-	27
120	(i)	82	81	28	24	36	34	38	38
	(ii)	-	74	33	25	-	31	-	36
150	(i)	107	105	49	45	56	56	61	56
	(ii)	-	87	47	39	-	46	-	51
180	(i)	108	110	83	81	83	85	105	95
	(ii)	-	100	76	71	-	76	-	87
210	(i)	99	99	133	141	118	115	136	136
	(ii)	-	113	122	126	-	125	-	133
240	(i)	98	105	154	165	144	149	140	145
	(ii)	-	115	168	187	-	175	-	178
270	(i)	87	91	155	171	162	183	153	169
	(ii)	-	102	176	199	-	183	-	173
300	(i)	95	104	157	172	157	178	149	167
	(ii)	-	88	138	148	-	140	-	127
330	(i)	78	85	97	96	79	77	78	77
	(ii)	-	76	89	86	-	88	-	80
Calms	(i)	15	-	16	-	15	-	11	-
	(ii)	-	-	(a)	-	-	-	-	-

(a) Absorbed in the lowest speed category.

In the "spring" there are two maxima for (F) - at 300° and, more markedly, at 180°. The variation of (D) is more irregular, with maxima at 180°, 240° and 300° : the lowest minimum for both parameters lies in the 90° sector as in the other seasons.

On a per mille basis, the values of (F) and (D) for a particular season and direction are approximately equal, and this agreement within a season/direction category implies that the mean speed of the wind differs relatively little with direction.

The theoretical values for (F) and (D) necessarily display only one maximum and only one minimum in any particular season, these occuring respectively in the sector containing the vector mean and in the diametrically opposed sector. It will be noted that, in this respect, the actual and theoretical distributions are in approximate conformity except in the "spring" when the theoretical maximum occurs in the 240° sector.

In "summer", "autumn" and "winter" the observed (D) tends to fall below the theoretical value in the 240° and 270° sectors with compensating excess in the 300° sector.

(F) and (D) for winds with an easterly component are not strikingly below theoretical expectation.

The polar diagrams show a persistent tendency for the mean speed from easterly points - "spring" excepted - to be greater than the theoretical value, and to be less in the 210°, 240° and 270° sectors.

The proportionate distribution of frequencies for the two partial series (1950-52, 1953-55) show - except for "spring" - a single maximum and minimum, but not necessarily in the same sector in the two periods. In "spring" the maxima for (F) in both partial series occur in the same two sectors, viz. 180° and 300° - this feature reappearing in the complete series.

(b) The frequency density field.

In all seasons the diagrams reveal:

A deficit - below the NCD value - of light winds.

An excess of winds of moderate strength, with one concentration somewhat north of west and a second to the south or east of south.

The 240° radius running through regions of small but persistent frequency deficits.

The actual and theoretical density contours coinciding more closely away from, than near to, the vector centre.

The major axis of the momental ellipse running from the south-east to the north-west quadrant linking the two centres of concentration mentioned above.

The tendency towards a bi-modal distribution, with anomalously high concentrations of wind in the south-east and north-west quadrants, strongly suggests a link with the wind sequence of depressions of middle latitudes.

(c) The distribution of displacement.

The data for "summer" have been analysed in detail. Relative to the frequency density diagrams the emphasis is shifted from light winds to the stronger winds, and it can be argued that the more important features of the two-dimensional patterns are thereby brought out.

The two local frequency maxima at about 200° and 300° are confirmed, and the excess in the south-east and north-west sectors is found to extend to winds of Force 6 or greater. A deficit in winds of all strengths from the east is revealed, as also that of stronger winds in a wide sector from 200° to 290°.

(d) The non-dimensional parameters.

Seasonal values of the parameters are given in Table 9(2):

<u>Table 9(2)</u>. <u>Actual values on a seasonal basis of q and $s/\overline{V}_{(s)}$, together with;</u>

(i) actual values of $\sigma/|\overline{\mathbf{V}}|$

(ii) estimated values of $\sigma/|\overline{\mathbf{V}}|$ appropriate to a normal circular distribution of parameter q.

Period and Parameter.			Season			
			Spring	Summer	Autumn	Winter
1950-52	q (%)		14.1	53.5	42.7	52.6
	$\sigma/\lvert\overline{\mathbf{V}}\rvert$	(i)	7.66	1.83	2.41	1.81
		(ii)	7.85	1.85	2.42	1.87
	$s/\overline{V}_{(s)}$		0.489	0.496	0.493	0.425
1953-55	q (%)		22.5	50.0	50.0	38.8
	$\sigma/\lvert\overline{\mathbf{V}}\rvert$	(i)	4.82	1.98	1.93	2.69
		(ii)	4.91	2.01	2.00	2.71
	$s/\overline{V}_{(s)}$		0.475	0.479	0.433	0.489
1950-55	q (%)		16.7	51.4	46.2	44.5
	$\sigma/\lvert\overline{\mathbf{V}}\rvert$	(i)	6.55	1.94	2.17	2.26
		(ii)	6.67	1.94	2.22	2.32
	$s/\overline{V}_{(s)}$		0.482	0.488	0.470	0.456

In all seasons but "spring" the values obtained for q must, it is suggested, be considered high for a surface wind distribution. The fair "within season" agreement for the three periods is worthy of mention.

In respect of particular months of given name, July returns consistently high values of q viz. > 60%, although other months, (December, August and September), give values of this order for one or other of the three-year periods. A low value of q appears to be a definite feature of the months of March and May.

Certain synoptic implications of the seasonal variability of q suggest themselves, but it is not proposed to examine this topic here.

As regards $\sigma/|\bar{V}|$, the interest is mainly in the difference between the actual and theoretical values of this quantity. On the three-year basis, the actual values for months of given name, range between about 1.50 and 3.50, the exceptions being March and May in both 1950-52 and 1953-55. In practically all series and sub-series, (including those for 0300 and 1500GMT)

$$\left[\sigma/|\bar{V}|\right] > \left(\sigma/|\bar{V}|\right)$$

but, except in the months of March and May there is little divergence on the basis of two significant figures.

It will be recalled that the $s/\overline{V(s)}$ ratio for an NCD is very nearly constant at 0.523 within the range $0 < q < 50\%$, dropping only slightly to 0.500 at 65%. The lowest actual value on a seasonal basis is 0.425 for "winter" (1953-55) when $q = 52.6\%$. For months of given name the highest values ("all hours" series only) are:

April, 1950-52, 0.510 when $q = 47.6\%$

August, 1950-52, 0.510 when $q = 64.9\%$

In the second case the actual value exceeds the theoretical (0.497), but otherwise in all series and sub-series the value for the appropriate NCD is greater. The lowest value ($s/\sqrt{(s)}$ = 0.369, q = 54.5%, December, 1950-52) is much below that for the most dispersed of the hypothetical distributions (See Fig. 6(2)).

In the light of the properties of the hypothetical distributions dealt with in Chapter 6, the divergencies (August 1950-52 excepted), of the values of both $\sigma/|\bar{V}|$ and $s/\sqrt{(s)}$ from those of the appropriate NCD, are consistent with a degree of dispersion for the actual distribution greater than that for the theoretical model. Experience suggests, however, that the dispersion probably arises from the slight bi-modal character of the pattern rather than from simple scatter.

(e) Some provisional conclusions.

The OWS "J" data has been examined at some length in order to reveal the character of the two-dimensional wind field for a station free from topographical obstruction; thus we may attribute characteristics of the field to synoptic influences.

On the basis of a sample for seasons extending over a six-year period, the following conclusions are tentatively advanced:

In respect of the gross parameters, viz. totals of frequency and displacement according to direction, mean speeds in given directions, q and $\sigma/|\bar{V}|$, the distribution for three of the seasons i.e., omitting "Spring", may be considered reasonably in agreement with an appropriate NCD of parameter q . In particular the actual fields of (F) and (D) give but one maximum, this and the single minimum lying in diametrically opposed sectors which are nearly the same ones in all three seasons, and with the vector mean located in, or in the sector adjacent to, the one containing the maximum.

In "spring" there is strong evidence of the occurrence of a greater variety of wind systems than in the other seasons, resulting in a multi-modal directional field for (F) and (D); largely as a result, the vector mean does not lie in the sector of maximum (F) and (D); thus a low value for q and a high one for $\sigma/|\bar{V}|$ is to be anticipated. Nevertheless, the appropriate NCD may, for some purposes be considered to represent the salient features of the distribution quite satisfactorily (it may be noted that the low q is also typical of "spring" at both Bell Rock and Lerwick).

Attention to the finer features of the pattern reveals, for all seasons, a local concentration of (F) and (D) in the north-west quadrant and a second concentration lying in a sector confined approximately within the range $120^\circ - 210^\circ$: these features, strongly suggestive of the wind sequences of depressions, are thus to be regarded as characteristic of synoptic variability. The bi-modal pattern gives rise to a deficit of (F) and (D), noticeably in the 240° sector.

Although relatively few in number, winds from an easterly point do not appear to be grossly under-represented, and except in "spring" the mean speed of wind in the easterly quadrants are high compared with the NCD value.

Except in "spring", the general results for the two three-year samples are found to be in fair agreement with each other. Thus the approximate regularity of the patterns for larger samples is not merely due to the elimination of casual variability by the increase of sample size. For "spring", however, there is a large difference between the vector means for the two three-year components, the month of May being primarily responsible.

Section 3. The "Northice" series.

(a) The distribution of frequency (F) and displacement (D) by direction.

The basic data ~~is~~ are set out in Table 9(3) shown on the following page.

In all seasons the actual values of (F) and (D), the theoretical value of (D) and, almost certainly also of (F), reach a well marked maximum in the 270° sector, but in "summer" and "winter" there is a weak subsidiary maximum centred at 120°.

Except in the "spring", there is a higher concentration of (D) in the 270° sector than is required theoretically, with a compensating tendency in the 240° sector.

The contribution to both (F) and (D) in the 30°, 60° and 90° sectors is very small, the absolute minimum lying in the 60° and not, as required by symmetry, in the 90° sector.

As might be anticipated mean wind speeds in the poorly represented section are rather erratic, but in all seasons there is a definite trend for them to be much in excess - often by a factor of two or more - of the theoretical value.

(b) The frequency density field.

The density contours fall into two portions; in the larger, situated around and to the west of the vector mean, the contours approximate to circular or somewhat elliptical curves, with a narrow nose stretching eastwards towards, and beyond, the origin, (this second feature is required to accommodate the occasional strong winds from easterly points).

Table 9(3). Frequencies (F) of surface wind observations, and of displacement (D); by direction in intervals of 30°, centred as indicated. "Northice", (November 1952 to June 1954).

(i) observed values;
(ii) values for a normal circular distribution, of parameter q derived from the actual series.

Central Direction (degrees).		Spring		Summer		Autumn		Winter	
		F	D	F	D	F	D	F	D
		(In parts per thousand).							
360	(i)	3	2	15	10	9	8	8	3
	(ii)	-	5	-	30	-	6	-	4
030	(i)	4	1	13	11	9	7	6	4
	(ii)	-	..	-	9	-	..	-	3
060	(i)	1	0.5	9	7	4	1	4	3
	(ii)	-	..	-	6	-	..	-	3
090	(i)	-	..	15	16	-	..	11	10
	(ii)	..	..	-	5	-	..	-	-
120	(i)	3	0.4	37	34	6	3	20	19
	(ii)	-	..	-	6	-	..	-	2
150	(i)	1	1	28	23	6	6	8	6
	(ii)	-	..	-	10	-	..	-	5
180	(i)	1	0.2	24	15	11	4	15	11
	(ii)	-	5	-	26	-	5	-	16
210	(i)	25	24	54	41	21	21	51	56
	(ii)	-	14	-	76	-	33	-	86
240	(i)	187	204	139	125	189	200	213	238
	(ii)	-	186	-	203	-	214	-	294
270	(i)	541	566	386	444	493	509	540	543
	(ii)	-	590	-	318	-	495	-	399
300	(i)	211	193	201	225	223	214	104	95
	(ii)	-	186	-	221	-	214	-	155
330	(i)	17	8	41	49	30	27	11	10
	(ii)	-	14	-	90	-	33	-	33
Calms	(i)	6	-	38	-	0	-	9	-
	(ii)	-	-	-	-	-	-	-	-

In all seasons there is an excess above the NCD value in the central portion of the field; in "summer" and "winter" this positive anomaly continues westwards from the mean centre; in "autumn" and "spring" it extends eastwards through the origin, leaving a deficit in the mid-speed ranges to the west. Wind frequencies from the north in "summer" and the south in "winter" are below the expected values.

(c) The analysis of displacement has not been carried out for this series.

(d) The non-dimensional parameters.

Table 9(4). Seasonal values of q and estimates of the parameters $\sigma/|\bar{V}|$ and $s/\sqrt{(s)}$:

(i) as obtained from the actual observation

(ii) as appropriate to a normal circular distribution of parameter q.

Parameter		Season: Spring	Summer	Autumn	Winter
$q(\%)$		93.8	76.4	89.9	85.8
$\sigma/\lvert\bar{V}\rvert$	(i)	0.583	1.043	0.635	0.745
	(ii)	0.50	1.10	0.655	0.78
$s/\sqrt{(s)}$	(i)	0.424	0.468	0.366	0.370
	(ii)	0.324	0.472	0.390	0.424

The outstanding feature is the exceptionally high values taken by q. Clearly there must be some marked physical control operating. Hamilton's discussion of the climate at "Northice" (Hamilton, 1958(a), (b)), leaves little room to doubt but that the flow is dominated by a katabatic wind generated by the extensive snow-covered surface which slopes gently down from the ice-cap towards the east. This basic flow is frequently interrupted by winds associated with cyclonic disturbances - Hamilton reports as many as 15 in some months - and almost certainly it would be these which give rise to the occasional strong winds from easterly points.

The actual values of $\sigma/|\bar{V}|$ run slightly below the theoretical ones except in the "spring". This exception could imply a degree of dispersion less than for the NCD, and scrutiny of the frequency density diagrams shows not only the high concentration of values near the mean centre, but also, relative to other seasons, a lack of entries remote from the centre. With q taking values much in excess of 50%, $s/\bar{V}(s)$ for the NCD is no longer constant, and, except in "spring", the theoretical values are greater.

(e) Some provisional conclusions.

There is evidence that this wind system is one in which a constant flow, largely induced by the properties of the surface, is frequently disturbed by passing cyclonic disturbances; but not to such an extent as to invalidate a model in which the synoptic influence is regarded as a secondary one. The resulting directional distribution of (F) and (D) is, on a seasonal basis, strongly uni-modal, but the relatively infrequent winds from the east give rise to mean wind speeds for these directions, of the same order as for westerly winds.

Section 4. Bell Rock series.

(a) The distribution of frequency (F) and displacement (D) by direction.

The data to be considered are set out in Table 9(5) on the following page.

Table 9(5). Frequencies (F) of surface wind observations, and of displacement (D); by direction in intervals of 30°, centred as indicated.
Bell Rock, December 1950 - November 1955.

(i) observed values;
(ii) values for a normal circular distribution, of parameter q derived from the actual series.

Central Direction (degrees).		Spring		Summer		Autumn		Winter	
		F	D	F	D	F	D	F	D
		(In parts per thousand).							
360	(i)	39	43	29	23	33	37	54	52
	(ii)	–	78	–	74	60	54	–	60
030	(i)	50	56	65	78	40	44	74	78
	(ii)	–	71	–	67	45	37	–	37
060	(i)	98	119	80	93	40	43	31	34
	(ii)	–	68	–	59	38	29	–	29
090	(i)	74	76	65	53	20	23	22	22
	(ii)	–	67	–	59	37	28	–	28
120	(i)	77	58	62	46	15	19	34	33
	(ii)	–	70	–	66	45	36	–	33
150	(i)	76	56	75	61	65	59	31	31
	(ii)	–	77	–	74	58	50	–	45
180	(i)	96	103	101	90	118	116	58	66
	(ii)	–	84	–	89	81	81	–	74
210	(i)	88	80	82	85	100	103	93	91
	(ii)	–	95	–	100	120	127	–	117
240	(i)	75	80	107	112	104	116	115	118
	(ii)	–	100	–	110	150	166	–	161
270	(i)	112	131	145	179	238	215	199	215
	(ii)	–	101	–	111	154	172	–	182
300	(i)	110	125	107	136	164	155	173	164
	(ii)	–	100	–	100	125	134	–	140
330	(i)	68	73	43	44	63	70	103	95
	(ii)	–	89		91	87	86	–	93
Calms	(i)	37	–	39	–	10	–	13	–
	(ii)	–	–	–	–	(a)	–	–	–

(a) Absorbed in the lowest speed category.

In all seasons the major maximum for both (F) and (D) occurs in the 270° sector, which also contains the vector mean. A well-defined secondary maximum occurs around 60° in "spring" and "summer", and around 30° in the other two seasons. A further maximum is located in the 180° sector in all seasons but "winter", and is most evident in the "autumn"; in "summer" this singularity appears to be due to a deficit in the adjacent 210° sector.

In all seasons, "winter" least so, a local minimum occurs in the 360° sector.

In all seasons there appears a deficit of (D) and of (F), in either or both of the 210° and 240° sectors.

In "spring" and "summer", mean speeds exceed their theoretical value in the two northern quadrants, and generally fall below expectation in the south-east, and to some extent, in the south-west sectors. In "autumn" and "winter" speeds are considerably in excess of the expected values in the two eastern quadrants, and below in the western ones.

In contrast to the findings for the stations considered in Sections 2 and 3, (F) and (D) at Bell Rock do not assume a uni-modal form, and, whilst the major maximum is to be found in the 270° sector, certain additional features persist; viz. a local maximum at 30° or 60°, a minimum at 360°, and a weak maximum at 180°. In respect of (D) the main maximum, the subsidiary maximum in the north-east quadrant and the minimum at the 360° reappear in the sub-series for 0400 and 1600GMT.

(b) The frequency density field.

The relevant diagram for "autumn" is reproduced as Fig. 7(7) on page 116.

In all seasons, a positive anomaly is to be found in the 30° and 60° sectors, mainly associated with stronger winds from an easterly point - in fact there is a deficit of the lighter winds in "autumn" and "winter".

In all seasons, a positive anomaly occurs in the south, and, except in "summer", there are occasions when the wind exceeds 25 knots from this direction.

A marked positive anomaly occurs in the 270° and 300° sectors, being least apparent in "spring".

In all seasons there is an appreciable deficit to the north, below the NCD values.

(c) The distribution of displacement.

The "autumn" diagram is reproduced as Fig. 7(8) on page 119, and some comments have already been made. In that diagram the existence of two centres of concentration south and west of the origin, the tendency for relative deficiency of (D) from the south-west, and the positive anomaly in the north-east quadrant, are clearly shown.

(d) The non-dimensional parameters.

Table 9(6). Actual values on a seasonal basis of q and $s/\bar{V}_{(s)}$, together with;

(i) actual values of $\sigma/|\bar{V}|$,

(ii) estimated values of $\sigma/|\bar{V}|$ appropriate to a normal circular distribution of parameter q.

Parameter		Season			
		Spring	Summer	Autumn	Winter
$q(\%_{10})$		10.4	22.5	42.7	43.0
$\sigma/|\bar{V}|$	(i)	11.04	5.02	2.41(5)	2.42(2)
	(ii)	10.15	4.90	2.43	2.42
$s/\bar{V}_{(s)}$		0.592	0.574	0.497	0.517

The magnitude of q is consistently less than for the OWS data and, in contrast to them, the "summer" value is low and not high compared with those for other seasons.

The frequency density pattern revealed in Fig. 7(7) shows important deviations from the NCD, yet it will be noted from Table 9(6) that the actual and theoretical values of $\sigma/|\bar{V}|$ agree closely and lie well within Brooks' tolerances (Fig. 5(3)).

Since $q < 50\%$ the NCD value for $s/\overline{V(s)}$ is uniformly 0.523, and divergencies from this value are positive for the two lower values of and negative for the two higher values.

(e) Some provisional conclusions.

The main peculiarities of the wind field, viz. relatively high (F) and (D) from the west, the north-east and the south, and persistent low values from the north and south-west, are not inconsistent with a crude model in which the adjacent natural features (see sketch map of Fig. 8(1)) operate as a system of mechanical constraints to a uni-modal flow pattern rather similar to that experienced at OWS "Juliet".

Considered in isolation, it would be unjustified to ascribe the deficit in the south-west to any mechanical influence exerted by the Fife peninsula, since the OWS series reveals a relative deficit in this sector also, although, for the marine winds, localised concentrations embedded in the two-dimensional fields are not reflected in the partitioning of (F) and (D) by direction. Whilst diurnal influences are important in "summer" (Goldie 1935), and judging from the behaviour of the vector mean, also in "spring", yet the tendencies provisionally ascribed to mechanical factors are evident in all seasons.

Marshall's (1954) comment on the Bell Rock winds may be recalled, (see page 49), in which he suggests that certain characteristics of the winds at this station are linked with the formation of small depressions which form off the coast when a general north-westerly air stream blows over the mountains of Scotland; this could well contribute to the deficiency in the northerly flow and possibly also to the excess from the west.

Section 5. The Lerwick series.

(a) Distribution of frequencies (F) and displacement (D) by direction.

The data for examination are set out in Table 9(7) shown overleaf.

In all seasons, (D), and except in "winter", (F), assume maximum values in the 180° sector, and in "autumn" and "winter" this is the absolute maximum for (D); these values are flanked by relatively high values in the adjacent 150° sector. Other maxima occur as indicated:-

at 270° - (F) and (D) in "spring" and "summer" (when the absolute maximum).
(D) in "autumn"

at 240° - (D) in "winter" and (F) in "autumn"

at 210° - (F) in "winter"

at 300° - (F) in "winter"

at 330° - (F) in "autumn"

at 360° - (F) and (D) in "spring", (D) in "winter".

at 030° - (F) and (D) in "summer".

Entries for the last three mentioned sectors might plausibly be considered as expressions of a common factor.

The extreme minimum, which is particularly well defined, occurs in the 90° sector in "spring" and "summer" and in the 60° sector for the other two seasons.

Table 9(7). Frequencies (F) of surface wind observations, and of displacement (D); by direction in intervals of 30°, centred as indicated.
Lerwick, 1951-55.

(i) observed values;
(ii) values for a normal circular distribution, of parameter q derived from the actual series.

Central Direction (degrees).		Spring		Summer		Autumn		Winter	
		F	D	F	D	F	D	F	D
		(In parts per thousand).							
360	(i)	102	101	78	87	97	90	101	98
	(ii)	-	80	-	86	-	54	-	58
030	(i)	72	78	96	103	29	26	40	45
	(ii)	-	71	-	64	-	45	-	44
060	(i)	72	72	42	38	13	11	17	16
	(ii)	-	66	-	52	-	42	-	40
090	(i)	38	31	36	27	25	20	20	17
	(ii)	-	62	-	45	-	46	-	41
120	(i)	61	57	40	30	59	75	35	35
	(ii)	-	67	-	45	-	58	-	50
150	(i)	90	76	89	67	112	120	71	91
	(ii)	-	73	-	53	-	77	-	67
180	(i)	94	99	109	89	134	129	114	138
	(ii)	-	82	-	58	-	107	-	94
210	(i)	85	94	67	68	86	88	123	121
	(ii)	-	93	-	92	-	133	-	123
240	(i)	86	107	93	127	105	123	112	126
	(ii)	-	104	-	120	-	142	-	155
270	(i)	96	114	126	145	104	128	99	109
	(ii)	-	108	-	138	-	127	-	139
300	(i)	79	85	99	120	96	97	113	117
	(ii)	-	103	-	134	-	99	-	111
330	(i)	90	86	90	99	106	93	108	87
	(ii)	-	91	-	113	-	70	-	78
Calms	(i)	35	-	35	-	34	-	47	-
	(ii)	-	-	-	-	-	-	-	-

Between the southern and western maxima there is, "winter" excepted, a persistent and quite definite minimum at 210^{o}.

The observed values of (D) to the north and to the south of the origin, definitely exceed the expected values, especially in "autumn" and "winter", but this is not the case with the less clearly defined western maximum.

As with the previous two stations, the mean speed of winds from easterly points tends to be greater, those from westerly points less, than given by the characteristic circle.

The general pattern presented by the polar diagrams for this station, is definitely elongated along the north-south axis, in contrast to the east-west orientation of the corresponding pattern for Bell Rock - see for example Fig. 7(6).

The vector mean is located within the 270^{o} sector which, in "autumn" and "winter" is <u>not</u> the one containing the highest values of (F) and (D). This feature constitutes an important difference from the behaviour at the other two stations.

(b) The frequency density field.

The main impression is that the high values of (F) from north and from south are due to an excess of winds of all strengths. To the east there appears some deficiency in winds of most speed levels, but it is less marked for the higher speed ranges than for the lower.

In all seasons there is a deficit of westerly winds at 7 metres per second or less. There is a local maximum to the south, as with the other stations, but in contrast there is no persistent tendency for a concentration in the north-west quadrant.

When preparing the diagrams, isopleths were not drawn in the immediate vicinity of the origin, but, in this region there is a marked, narrowly based, peak in the frequency surface due to a proportion of calms much in excess of that proper to the NCD.

(c) The distribution of displacement.

The most marked feature of the diagram is the emphasis given to the deficiency in (D) from westerly points with winds of 10 metres per second or less: at higher speeds there is a degree of compensation.

The deficiency in flow from the east appears most marked in "autumn". In all seasons the positive anomaly occupies a wide ring broken towards the east over a directional span of about 45°. In three seasons there is a region of concentration south of the origin, but no persistent evidence of a second to the north-west, a feature responsible for the bi-modal appearance noted in the other two series.

(d) The non-dimensional parameters.

Table 9(8). Actual values on a seasonal basis of q and $s/\bar{V}_{(s)}$, together with:

(i) actual values of $\sigma/|\bar{V}|$,

(ii) estimated values of $\sigma/|\bar{V}|$ appropriate to a normal circular distribution of parameter q.

Parameter		Season			
		Spring	Summer	Autumn	Winter
q(%)		15.1	28.2	29.8	32.7
$\sigma/\lvert\bar{V}\rvert$	(i)	7.63	4.09	3.76	3.43
	(ii)	7.40	3.90	3.65	3.30
$s/\bar{V}_{(s)}$		0.593	0.637	0.584	0.619

Only in "winter" does q exceed 30 percent, hence this quantity runs at a lower level than is the case for any of the other three stations.

Comparisons between estimates suggest the possibility of a distribution more concentrated than the NCD, and this conclusion is supported by the fact that the values of $^{s}/_{V(s)}$ appreciably exceed the 0.523 appropriate to an NCD for $q < 50\%$. A tentative explanation is that the relatively high proportion of calms and the natural tendency for the concentration of vector ends at the mean centre - the distance of which from the origin is small compared with the range of speeds reported - give rise to a highly concentrated field. It would now appear reasonable to assume that a high value of $^{s}/_{V(s)}$ is a characteristic of land stations.

Section 6. Some general conclusions.

Before appraising the results it is desirable to recall some details of the sampling procedure. For each of three of the stations, ("Northice" excepted), data for some 15 or 18 months were aggregated to form the basic tabulation; observations for months of given name for five or six years being combined to give seasonal batches. For Bell Rock and Lerwick a satisfactory surface wind distribution was presumed to be obtained by pooling twice daily observations. For the marine data there was a 24 hour cover, although there was evidence that an analysis basis based on either the 0300 or 1500GMT observations might be satisfactory.

Synoptic variability as expressed by the OWS data suggests the following properties of the two-dimensional fields:

(i) The directional distribution of (F) and (D) is uni-modal, the maximum for both quantities lying in the sector containing the vector mean and the minimum in the diametrically opposed sector.

(ii) A finer analysis reveals, in both the (F) and (D) fields, certain centres of local concentration, mainly in the south-east and north-west quadrants - features which may plausibly be linked with the sequence of winds in depressions.

(iii) As a theoretical model the NCD would seem to be both valid and useful, although it must be noted that the features listed in (i) and (ii) are quite apparent, and are merely more clearly delineated, not initially revealed, by comparison with the theoretical model.

(iv) The normal circular distribution requires that mean speed in any given direction should decrease with increasing angular divergence from the vector mean. The observed mean speeds from easterly points, especially between north and east, are, however, greater than theoretically required, and this implies that for each directional category (F) and (D) are more nearly proportional than required theoretically. It thus follows that the partitioning of frequency by direction is, for a freely exposed station, a closer guide to the distribution of displacement than would be produced by the NCD.

An associated feature is that strong winds are by no means confined to the sector containing the highest (F).

Familiarity with handling the data points to the possibility that in the more remote parts of the two-dimensional field there is a greater measure of regularity than nearer the vector mean - actual and theoretical contours running together. This prompts the suggestion that the analysis of wind fields according to wind speed strata may be profitable.

We would expect that the characteristics of the wind field due to mechanical constraints would be more clearly evident in those months of the year when diurnal influences - especially land and sea breezes on a smaller scale, or on the larger scale as discussed by Goldie in respect of Bell Rock - would be least evident. Our analysis suggests that such diurnal influences are to be expected in the "spring" months, viz. March, April and May as well as in June, July and August mentioned by Goldie.

The actual values obtained for $\sigma/|\bar{V}|$ rarely fall outside the limits proposed by Brooks (this was true even for certain hypothetical distributions): in fact, we may conclude that Brooks was in error to allow such wide tolerances and it has been shown, e.g., Tables 7(5) to 7(8), that quite small differences in $\sigma/|\bar{V}|$ have led, through the consequential changes in σ to alternative distributions which may well be considered to differ significantly. This point is emphasized by an examination of the second parameter $S/V(S)$ which appears very sensitive to deviations from the NCD.

The "Northice" series presents us with an example in which synoptic factors may be considered to be subsidiary to mechanical ones in the generation and maintenance of the wind system. The series shows that it is possible for surface wind patterns to be highly concentrated - as evidenced by the high value of q. The two-dimensional field is, on the whole uni-modal. The occasions of relatively strong winds from easterly points would appear to be a manifestation of synoptic influences. As with the marine winds, there is evidence that mean speeds from easterly points are greater than required theoretically, and that the actual and the NCD frequency density contours agree more closely in regions remote from the mean centre than near to it.

Observations from Bell Rock and Lerwick were selected for study on the assumption that the anemometer at these two well-exposed stations in Scotland, would be influenced to the minimum extent by buildings and similar structures, would be free from the effects of very localised features of ground contour and, it was considered, free from near-by topographical features of horizontal and vertical dimensions of the order of 10^3 and 10^2 feet respectively which, almost invariably in the British Isles, are found to influence the readings of even the best sited anemometers. On general grounds some effects due to "mechanical" constraints were anticipated, but only after the marine winds were analysed was it possible to attempt a semi- quantitative examination of winds from land-based stations.

Assuming provisionally that, over a period, both stations were subjected to the same general weather situations, we note a striking difference in the gross patterns of the (F) and (D) fields - those for Bell Rock being orientated about a long east-west axis, and those for Lerwick about a north-south axis. The tendency for a north-south orientation is, by itself, not convincing evidence of topographical constraints, since the OWS data reveals such a characteristic embedded in the frequency density field. Reviewing the evidence provided by the two stations jointly, and bearing in mind that the multi-modal fields of (F) and (D) are due to the persistence of certain deviations irrespective of season, it may be conceded that it is now possible to give a little more precision to those models of wind distributions for land-based stations, in which purely mechanical factors contribute towards the role played by natural topographical features.

Further studies are planned in which comparisons will be made between the wind at the surface and that measured more or less simultaneously at several hundred feet above it, and others in which attention will be paid to up wind surface "roughness" through the type of approach sketched in Appendix I(a).

Sufficient preliminary work has also been carried out concerning the momental ellipse and the orientation of the axes in respect to cardinal points, to the direction of the vector mean etc., to indicate that intensive study of these aspects would be of value.

Reviewing the results of the analyses for all four stations, the values of the non-dimensional parameters show increasing numerical divergence from the NCD values as we move from the "ideal" exposure of the Weather Ship through "Northice" and Bell Rock to Lerwick. These divergencies are linked with characteristics of the wind field which are, in broad terms, parallelled by the deviation for hypothetical distributions of varying degrees of concentration about the vector mean. The agreement, however, is not claimed to be more than sufficiently suggestive to encourage further investigation of the properties of idealised two-dimensional density distributions.

Tentative generalisations from the array of parameters are:

(i) For stations situated off the north-western seaboard of Europe, a low value of q (say $<$ 20%) is typical of "spring".

Well away from land, values for the other <u>seasons</u> lie within the range 45% - 60%, and values exceeding 60% (which do occur in means for months of given name) should be considered high for surface wind distributions.

For land-based stations the seasonal value for q ("spring" excepted), ranges from about 25% to 45%; values for the "summer" tending to be least - doubtless associated with the lower general level of wind speed, and the existence of large diurnal effects.

The relatively low values assumed by q are probably to be attributed to the existence of bi- or multi-modal distributions rather than to uniformity of scatter about the origin.

(ii) The differences between the computed value of $\sigma/|\bar{\mathbf{V}}|$ and that for the appropriate NCD lie well within the Brooks' tolerances and hence are numerically small; (in general the theoretical exceeds the actual value). Our analysis indicates, however, that by employing the slightly different values of σ, or $|\bar{\mathbf{V}}|$ or $\bar{V}(s)$ allowed by the tolerances, the resulting family of normal circular distributions are recognisably different judged in the light of marginal frequencies for "all speeds" and for "all directions" in addition to differences in any given speed/direction cell: Whether or not the divergencies between resulting NCD's are physically significant depends upon other considerations, but, nevertheless it is claimed that a useful first analysis of the type undertaken in this thesis is much aided by the employment of the NCD (of given q) as a theoretical model.

meaning?

(iii) The parameter $s/\bar{V}(s)$, which has been developed, is more sensitive than $\sigma/|\bar{\mathbf{V}}|$ to deviations of the actual from the theoretical distribution. It has been shown that for $q < 50\%$, the value of $s/\bar{V}(s)$ is nearly constant at 0.523, and it follows that for land-based stations the standard deviation of wind-speed is approximately half the mean speed for surface wind distributions which take the normal circular form, and conversely.

Especially in the "summer" half-year, it would appear that $s/\bar{V}(s)$ may exceed the expected value: this would seem to arise, not strictly from a high concentration about the mean centre, but from a combination of properties typical of land-based stations, viz. a relatively high proportion of "calms" and a value for mod $\bar{\mathbf{V}}$ small in comparison with σ.

Turning to the definition of "canalisation" or "funneling", certain necessary conditions may now be suggested viz:

On the average and on a seasonal basis local maxima of frequency and displacement must emerge.

These maxima should be evident in all seasons.

The presumption of canalisation is strengthened:-

(i) if the local maxima exceed the values of the appropriate NCD in the particular sectors in which they occur;

(ii) if these maxima emerge from analyses of observations for smaller samples, e.g. for individual seasons, for months of given name, etc;

(iii) if the local maxima do not occur in the sector containing the vector mean;

(iv) if (in some topographical situations) the maxima occur in pairs in diametrically opposed sectors.

If the general orientation of the pattern can <u>then</u> be satisfactorily associated with the topographical features - considered as mechanical constraints - it may be assumed that canalisation exists.

P A R T III

THE INTERCEPTION OF THE DIRECT SOLAR BEAM

BY A PLANE SURFACE

Chapter 10

A Method for Calculating the Angle of Incidence of the Direct Solar Beam on a Plane Surface of any Slope and Aspect.

The estimation of the radiation from sun and sky intercepted by a plane surface requires, inter alia, the calculation of the angle of incidence of the solar beam on the surface. Various methods have been described in the literature, e.g. by Brooks, F.A. (1951), Carruthers (1948), Kaempfert (1942), von Schedler (1950), Stagg (1950), Unna (1947), and an approximate graphical approach, in a publication of the Housing and Home Finance Agency, Washington (1954). It is now proposed to present a further method, which exhibits certain advantages of geometrical elegance and ease of calculation over the other techniques, and which permits the general application of a useful graphical device described by Schütte (1931).

After this work had been completed, the writer's attention was drawn to a paper published by Schütte (1943), in which the essentials of the method to be discussed had been set out. However, Schütte used a different coordinate system (viz. the celestial sphere) and did not explicitly show how the graphical device mentioned in his 1931 paper could be employed: in addition it can be argued that the alternative approach now to be described reveals more clearly the geometrical significance of certain expressions.

The customary assumptions for this type of problem have been made, viz. the earth is spherical, the difference between solar and siderial day may be neglected, refraction can be ignored, and the sun's declination may be regarded as constant during any particular solar day.

This last assumption is not absolutely essential, although without it Schutte's (1931) very useful technique cannot be applied.

Section 1. Method.

(a) The "corresponding" plane.

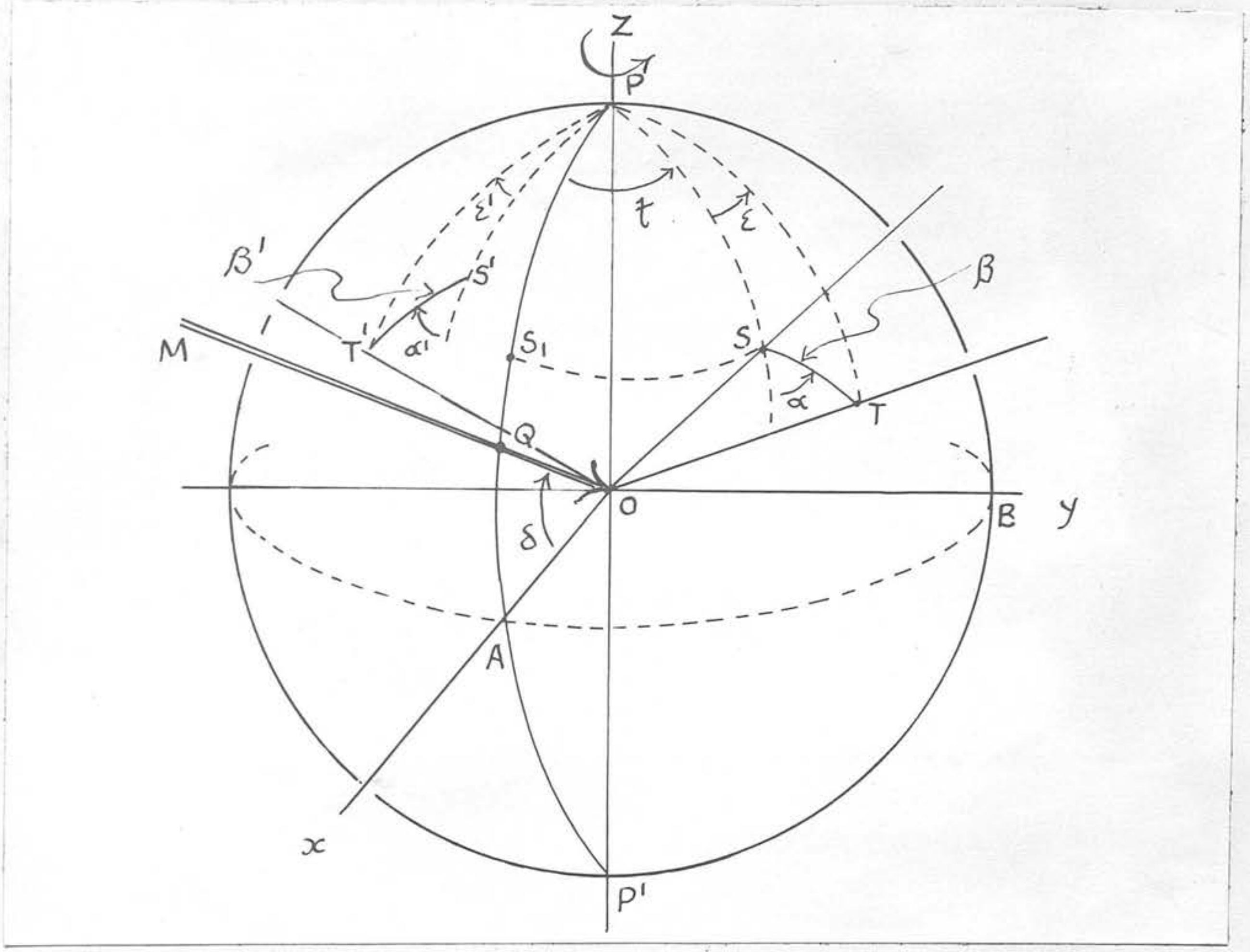


Figure 10(1). Coordinate system for calculation of angle between the sun's beam (MQO) and OT, OT'.. parallel to the normals to surfaces at S, S',.. of slope β, β',.. and aspect angle α, α'.

If, at any point S on the earth's surface Fig.10(1) there is a south facing slope of angle β to the horizontal, then, following Unna (1947) and

Carruthers (1948) a point T can be found, angular distance β southwards along the meridian through S, where a horizontal plane will be parallel to the sloping plane at S. More generally, if at S there is a plane of angle of slope and "aspect angle" α (α is the angle between the great circle containing the line of greatest slope and the plane of meridian through S) then T will lie arcal distance β along the great circle through S.

Points such as T, T^1, ... will (following Schütte) be termed "corresponding" points and tangent planes at T, T^1, 1.. "corresponding" planes. Certain simple geometrical properties of this system of corresponding points and planes are revealing; e.g. the corresponding plane for a vertical east or west facing surface lies at the equator whatever the latitude of the original point S.

The normals to the pairs of planes at S and T; S^1 and T^1.... will be parallel, making angles θ , θ^1,... with the sun's beam MQO where $\theta = \angle$ MOT, $\theta^1 = \angle MOT^1$... If I be the intensity of the solar beam, the flux normal to the sloping surface will be I cos θ ; thus we require cos θ in terms of declination δ, and hour angle t.

(b) <u>Coordinate System</u>

With O the centre of the earth as origin, let a right-handed set of axis be chosen, Ox (towards the sun at the equinox), Oz (through the pole P), and Oy. Suppose Ox remains fixed in space, then the line MO from the sun to O will lie in the xOz plane for all δ. α $(-180^\circ \leqslant \alpha \leqslant 180^\circ)$ is measured positive from south through east, and β $(0 \leqslant \beta \leqslant 90)$ is the inclination of the line of greatest slope with the horizontal.

For the horizontal surface at S the required angle θ (S) is $\angle$ MOS, and for the sloping surface it is θ (T) viz. $\angle$ MOT.

Imagine the earth to rotate in a positive sense within the spherical framework of which P^1 B P A P^1 forms a portion, then S will assume various positions and will be at S_1 at local apparent noon. Local apparent time, defined by hour angle t will be negative before local apparent noon (l.a.n.) and positive after l.a.n.

The point T is specified by ε (the difference in longitude between S and T), and by its co-latitude, defined by the arcal distance PT - for which the notation (PT) will be adopted in this work.

Clearly for the sloping plane at S, l.a.n. as "<u>apparent from the sloping plane</u>" will occur at $\varepsilon/15$ hours before l.an. for the horizontal plane at S and $\varepsilon'/15$ hours after in the case typified by S^1 and T^1.

(c) <u>Basic formulae</u>

In the spherical triangle PQT, if ϕ be the latitude of S and ψ ($= 90 - \phi$) its colatitude, then at hour angle <u>t</u> and declination δ

$$\cos\theta(T) = \sin\delta\cos(PT) + \cos\delta\sin(PT)\cos(t+\varepsilon) \qquad 10\,(1)$$

where from triangle PST

$$\cos(PT) = \cos\psi\cos\beta + \sin\psi\sin\beta\cos(180-\alpha) \qquad 10\,(2)$$

and

$$\cos\varepsilon = \frac{\cos\beta - \cos(PT)\cos\psi}{\sin(PT)\sin\psi} \qquad 10\,(3)$$

The sign of ε is the same as for α (viz. positive for slopes with aspect from south through east to north).

When $\beta = 0$ Eq.10(1) reduces to the familiar form

$$\cos\theta(S) = \sin\delta\sin\phi + \cos\delta\cos\phi\cos t \qquad 10\,(4)$$

In Eq. 10 (1) the quantities (PT) and ε are parameters defined by the geometry of the surface, whilst t and δ vary with time. The explicit occurrence of δ allows variations of θ(T) with time of year for a given surface to be readily studied: in contrast, the examination of variations of θ (T) for a given date but for different α, β may be more conveniently approached by von Schedler's formulation in which α, β occur explicitly.

For given ϕ and constant δ Eq.10(4) may be written

$$\cos\theta(S) = a + b\cos t \qquad 10(5)$$

and Eq.10(1) $$\cos\theta(T) = a' + b'\cos t' \qquad 10(6)$$

t' being the time from l.a.n. with respect to the sloping plane. Most advantages of this present formulation are direct consequences of the equivalence in form of Eq.10(5) and 10(6).

(d) A graphical aid by Schütte.

Schütte (1931) noted that by plotting on cos/cos graph paper, Eq.10(5) and 10(6) reduce to the form

$$y = a + bx$$

In practice the required lines are most conveniently defined by $\cos t = 1$; $\cos t' = \cos(t+\varepsilon) = 1$, at the corresponding l.a.n., and $\cos\theta(S) = \cos\theta(T) = 0$ at apparent sunrise or sunset.

Section 2 - Illustrative Examples

(a) Let $\phi = 55°19'$N. ie. $\gamma = 34°41°$.

$\alpha = 45°$ $\beta = 30°$ $\delta = 9°46'$N.

i.e. a south-easterly slope of 30° at Eskdalemuir Observatory (55°19'N., 3°12'N) in mid-April.

The under surface of this slope will be facing north-west, i.e. will have a north-westerly aspect and the appropriate values of $\cos\theta(T)$ for this case are given immediately by negative values from Eq.10(1).

Now $\alpha = +45°$, thus also positive.

From Eq.10(2) $\cos(PT) = .51096$; and from Eq.10(3) $\cos\varepsilon = .91156$ or $\varepsilon = 24°17'$.

Hence, $\cos\theta(T) = 0.08668 + 0.84710\cos(t+\varepsilon)$ (from Eq.10(1)).

and $\cos\theta(S) = 0.13950 + 0.56079\cos t$. (from Eq.10(4)).

When $t' = 0$ $\quad t = -\varepsilon = -24^\circ 17'$ $\quad \cos\,\theta(T) = .93378$ 10(7)

$t = 0$ $\quad \cos\,\theta(S) = .70235$ 10(8)

Again, when $\cos\,\theta(T) = 0$ $\quad t' = t + \varepsilon = \pm\, 95^\circ 52'$ 10(9)

when $\cos\,\theta(S) = 0$ $\quad t = \pm\, 104^\circ 24'$ 10(10)

These results are graphically displayed in Fig.10(2) (convenient scales for working purposes are 1 inch = 0.1 in $\cos\theta$ and $\cos t'$; this renders possible accuracy to two figures with very little error in the third figure).

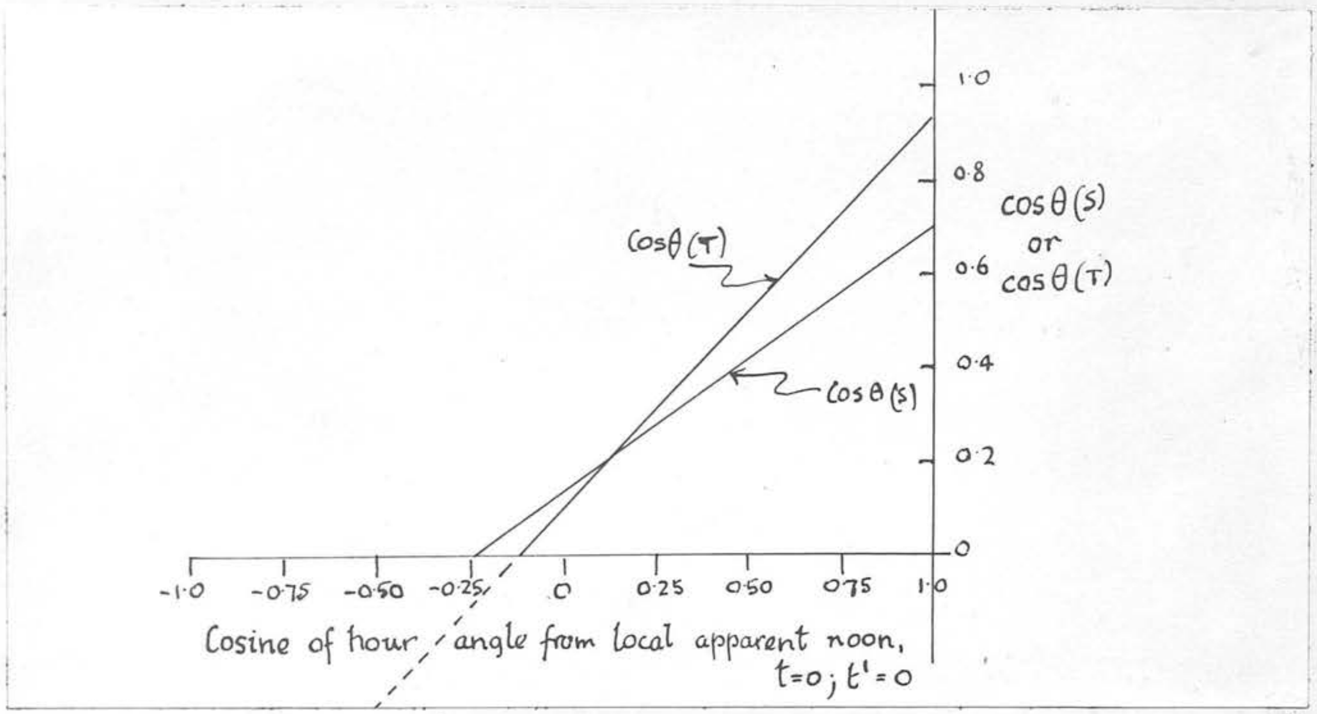


Figure 10(2). Cos $\theta(S)$ and cos $\theta(T)$ against cos t and cos t' respectively, where $\theta(S)$, $\theta(T)$ are the angles between the sun's beam and the normals to a horizontal and sloping surface (angle of slope $\beta = 30^\circ$, aspect angle $\alpha = 45^\circ$), and t and $t(= t + \varepsilon)$ local apparent time with respect to the two surfaces.
(Eskdalemuir Observatory (55° 19'N., 3° 12'W) solar declination 9° 46'N.)

One criticism may be anticipated at this point. For aspect angles $|\alpha| < 90^\circ$ and moderate β, slightly simpler expressions than Eqs. 10(2) and 10(3) are possible; in particular $\sin\varepsilon$ may be computed instead of $\cos\varepsilon$. However, owing to the ambiguity

$$\sin\varepsilon = \sin(180 - \varepsilon)$$

cos ξ is a better parameter. In fact, it was ambiguities of sign which led to the use, in all cases, of the slightly more complex, but more general, formulation described above.

For application it is necessary to relate cos θ(S) and cos θ(T) to local apparent time for the horizontal plane, as indicated in Fig. 10(3).

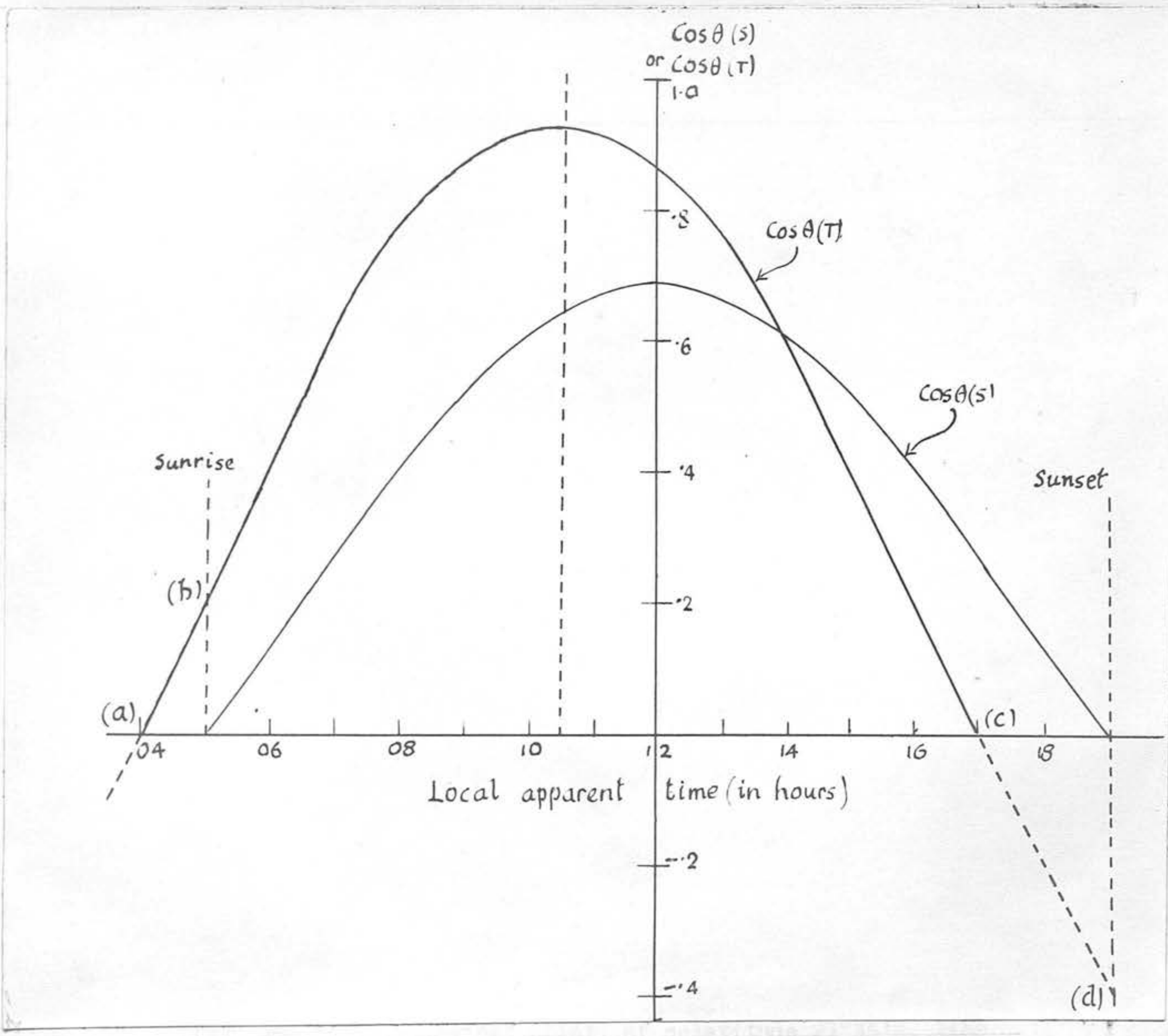


Figure 10(3). Cos θ(S) and cos θ(T) against local apparent time with respect to the horizontal surface, estimated graphically from Fig. 10(2).

(a) (b) Sloping surface not irradiated by sun; portion (a) (b) occurs before sunrise.

(c) (d) Negative values of cos θ(T): underside of surface irradiated by sun.

Typical points on the "T" curve are:-

$\cos\theta(T) = 0$; hence $t = -24^\circ17' - 95^\circ52' = -120^\circ9'$

or $t' = -24^\circ17' + 95^\circ52' = +71^\circ35'$

giving the points (a) and (c). If $\cos\theta(T) = .500$, then

$$\cos t' = \cos(t + \varepsilon) = 0.490 \quad \text{(from Fig.10(2))}$$

$$t' = \pm 60^\circ38', \quad t = -84^\circ56 \text{ or } 36^\circ22'$$

In practice $\cos\theta(T)$ and $\cos\theta(S)$ would be read off at each half hour l.a.t., as the data on radiation, sunshine and other elements likely to be studied in association with solar altitude are usually expressed in terms of quantities between exact hours.

(b) Let $\phi = 55^\circ19'N$, $\alpha = -150^\circ$, $\beta = 45^\circ$, $\delta = 23\frac{1}{2}^\circ N$

i.e. a slope of 45° to be horizontal and aspect 30° west of north at the summer solstice.

$\alpha = -150^\circ$ therefore ε will be negative.

$\cos(PT) = 0.92993$ and $\cos\varepsilon = -0.27512$ or $\varepsilon = -105^\circ58'$

Thus $\cos\theta(T) = 0.37081 + 0.33734 \cos t'$, and when $t' = 0$ we have $t = -\varepsilon = +105^\circ58'$ and $\cos\theta(T) = 0.70815$. This, the maximum value of $\cos\theta(T)$, will occur at about 18.45 l.a.t. (i.e. $1200 + 105^\circ58'/15$).

For $\cos\theta(T) = 0$ we have $\cos(t+\varepsilon) = -0.37081/0.33734$ which is inadmissible (see below (i) and (ii)), and for our second point we may choose $t = 90^\circ$, then

$$t + \varepsilon = -15^\circ58' \text{ and } \cos\theta(T) = 0.69514$$

Some features require comment:-

(i) The "corresponding" point, of colatitude $21^\circ35'N$, lies within the Arctic circle, thus when $\delta = 23\frac{1}{2}^\circ N$ the sun will remain above the horizon. Hence a negative value of $\cos\theta(T)$ will not occur.

(ii) If $\cos\theta(T)$ is put equal to zero an inadmissible value of $\cos t'$ follows: another point must be chosen, this is conveniently that for $t = 90^\circ$.

(iii) When using Eq. 10(2) we need to write $\cos(-180 - \alpha)$ giving $\cos(-180 + 150)$, since westerly aspects are measured by a negative rotation from south (see Fig. 10(1)).

(c) The final examples, illustrated in Fig. 10(4) are again for latitude $55^\circ 19'$N and $\delta = 23\frac{1}{2}^\circ$N, when $\alpha = 0$ or 180° and $\beta = 0, 30^\circ, 90^\circ$.

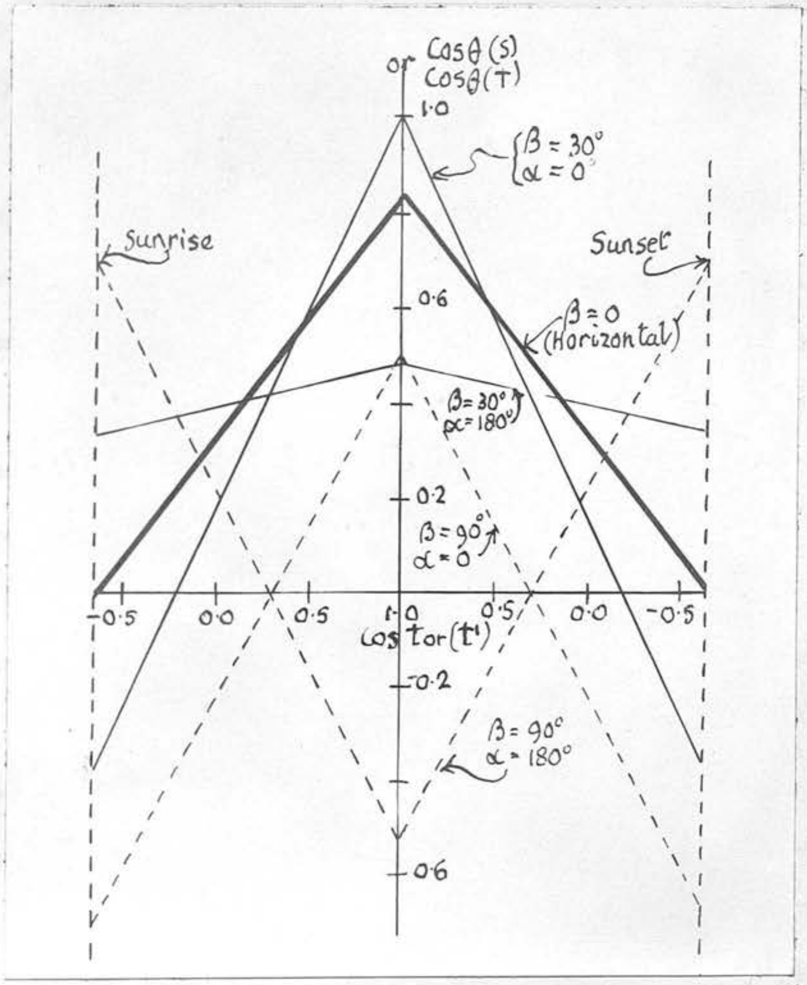


Figure 10(4). Values of $\cos\theta(T)$ for horizontal ($\beta = 0$) and for north and south facing slopes ($\beta = 30^\circ, 90^\circ$: $= 0^\circ, 180^\circ$) at $55^\circ 19'$N, $3^\circ 12'$W when $\delta = 23\frac{1}{2}$ N. (summer solstice).

The Schütte diagrams for all these cases may be superimposed, as the horizontal co-ordinates are all measured from the same zero.

In these cases there is, of course, no need to use Eq.10(1) to obtain the maximum ordinate as this follows from simple geometrical considerations (see Fig.10(1)). At the equinox (when $\delta = 0$) all lines will pass through the points on the abscissa corresponding to $t = 90^\circ$ or $\cos t = 0$.

Section 3 - Further Exploitation of the Technique.

Although a considerable amount of additional work has been carried out, details will not be presented in this thesis.

The investigations have included a comparison of various methods of computing $\cos\theta(T)$ as described by some of the authors mentioned in page 165; the partial preparation of working diagrams for two stations (Eskdalemuir and Lerwick Observatories) in order to reduce the arithmetical work involved in computing cos (PT) and ε ; and a trial computation has been made of $I \cos\theta(T)$ for a particular case. Note has also been taken of the advantages of seeking to express the intensity of the incident beam for specified sky conditions in the form

$$I = I_0 f(t)$$

If $f(t)$ can conveniently be expressed in terms of cos pt, (p an integer) then

$$\int_{t_1}^{t_2} I_0 f(t) \cos\theta(T) \cdot dt = \int_{t_1}^{t_2} I_0 f(t) [a' + b' \cos t'] . dt$$

is simply evaluated.

A preliminary study has also been made of the interception of diffuse radiation by a plane surface, in order to allow the computation of total short wave input on to sloping surfaces.

It will be realised that the low elevation sun experienced in such countries as Scotland, renders questions of radiant interception by sloping surfaces at least as important as in lower latitudes, and certainly so as regards agricultural and allied applications.

SUMMARY AND CONCLUSIONS

Chapter 11

Summary and Conclusions.

1. The need for comprehensive information on the characteristics of air flow over all types of terrain - from level grassland through arable areas, undulating park-land, forested regions to highly dissected mountain areas - has recently been underlined by the desire to rationalise procedures for reducing, by the provision of "shelter", certain weather stresses on crops and livestock, due to, or associated with, winds of moderate strength or greater (say $>$ 10 m.p.h.).

The meteorological conditions in which these practical problems arise are generally such as to permit the assumption of an adiabatic lapse rate.

2. Note has been taken of work in which the parameters, in the wind profile laws, relating to surface conditions have been tentatively extended to cover types of surface roughness having dimensions several orders of magnitude greater than the short turf, or similar surface, with respect to which the profile laws have been developed.

Examination of this work suggests:-

(i) that over areas carrying rigid, sharp-edged roughness elements, e.g. buildings uniformly distributed, the logarithmic law

$$\bar{u} = \frac{1}{k}\sqrt{\frac{\tau}{\rho}} \cdot \ln . \frac{z-d}{z_0}$$

may be useful, where values for z_0 and d (assumed or deduced) are plausibly associated with the linear dimensions of the roughness elements:

meaning?

(ii) that with obstacles which are wind deformable or of a lattice structure, e.g. as presented by an orchard, by brush country or by a forest, the extension of the above law in the form

$$\bar{u} = \bar{u}(0) + A\sqrt{\frac{\tau}{\rho}} \ln . \frac{z-d'}{z_0'}$$

($\bar{u}(0)$ = a threshold mean speed, and A , d' , z_0' possibly adjustable constants)

similar to that developed for flow over a moving sand-bed, may be of value.

3. Recent wind tunnel and field work, relating to the influence of shelterbelts and wind breaks has been reviewed.

(a) For single, artificial, two-dimensional barriers of known (geometrical) permeability ϕ , of sufficient cross-wind length (not less than about 20 times the height (H) of the barrier), standing on a horizontal surface and exposed to a steady wind of approximately perpendicular incidence, the "zone of influence" and the "efficiency" are known to a degree of accuracy adequate for many agricultural and horticultural purposes.

At a distance of about 20 H downwind from barriers of the type mentioned (cases of $\phi < 15\%$ excluded), the incident flow is largely re-established as judged by reference to the vertical profile of mean speed, to the mean direction, and to the finer eddy structure (such as revealed by records from a bi-vane of the Scrase/Best type exposed for periods of the order of one minute): with impermeable barriers, eddies of linear dimensions of order H pass through, and beyond, the 20 H limit. Complete re-establishment of the incident flow is attained at 30 H or so. These results probably apply also when the incident wind direction is oblique to the barrier, providing its projected length perpendicular to the wind is at least 20 H. When the cross-wind length is less than 20 H, the plan view of the protected zone assumes a rough parabolic form and the potential sheltering effect implicit in the height H is not fully realised. 2

Providing fully rough flow is ensured, field and model studies for artificial barriers of given ϕ are in good agreement. For low values of ϕ i.e. when very fine mesh is called for in the wind tunnel study (mesh size a millimetre or so), the similarity is prejudiced, probably because the effective aerodynamic permeability of the model is less than that of a full scale barrier. 2

(b) The general view is that with a system of barriers 30 H apart there is no cumulative effect, i.e. the zone aft of the final downwind barrier is no greater than for a single barrier of ϕ equal to that for the system of barriers: there is, nevertheless, an enhanced effect between successive barriers and some evidence of a lateral spread of the zone. Some workers have, however, advanced reasons for the view that if there is a sufficient number of barriers a cumulative effect will occur. Same?

(c) Similarity of flow between "live" shelterbelts and hedges, and their presumed model analogues, has not been attained to the same extent as for the rigid artificial barrier. This may be attributed to the difficulty of ascertaining a value for the permeability of the "live" barrier and of creating a model having that permeability. Nevertheless the agreement between model barriers (both artificial and live) and their full scale analogues is, when linear dimensions are expressed in terms of H, sufficiently satisfactory for quite a number of practical purposes, and it is found that the gross features of the mean flow pattern vary in a similar manner with ϕ .

4. As regards flow over undulating ground, evidence converges to indicate that the boundary layer will not leave the surface if the downward slope on the lee side is less than some ratio between one in five and one in ten. This leads to a point of considerable practical importance, viz. ground contour itself will not provide shelter from wind of moderate strengths or greater, in areas where the degree of slope is small enough to permit intensive arable and pastural farming: it is otherwise in the hilly areas suitable for livestock rearing.

5. Preliminary work indicated that some features of the two-dimensional field of surface wind might be clarified by using the normal circular distribution as a theoretical model.

The formal equivalence is noted between certain parameters of the wind distribution and certain of the coefficients of inertia of a two-dimensional system of point masses situated at the vector ends. In particular the standard vector deviation of the wind distribution corresponds to the radius of gyration, about the mass-centre, of the system of point masses.

Published methods of analysing upper wind fields with the aid of the normal circular distribution rest heavily upon an empirically determined, one-one relationship between q and $\sigma/|\bar{V}|$. This result has now been checked in considerable detail by the use of certain analytical functions and a tabulation recently published by the Rand Corporation. The empirically derived results were found to be exceptionally accurate.

A second parameter $s/\overline{V(s)}$ was utilised and a relationship deduced between it and q, σ, $|\bar{V}|$ from which it was found that, for a normal circular distribution, <u>when $q \leq 50\%$</u>

$$s/\overline{V(s)} \simeq 0{\cdot}52$$

Values of $q \leq 50\%$ were subsequently concluded to be typical of surface wind distributions.

6. The relationships between q , σ, $|\bar{V}|$, $\bar{V}(s)$ and s were then investigated for certain hypothetical density distributions which departed appreciably from the normal circular form, yet it was found that, in the main, the value of $\sigma/|\bar{V}|$ for a given q , differed numerically from that appropriate to the normal circular distribution by amounts lying within the published tolerances. It is concluded that the customary tolerances are too wide.

The parameter $s/\bar{V}(s)$ was found to be much more responsive to the differences in form between the several distributions.

7. Methods for dissecting, and for constructing, speed/direction frequency tables were examined, as were those for analysing "flow" or "displacement" (referred to as (D)).

Numerical and graphical techniques were developed for producing the frequency density distribution and the two-dimensional partitioning of (D) for a normal circular distribution of given q. Results obtained by the various methods and by independent checks were in very good agreement, differences being no greater than a few percent. Specimen examples are given of the diagrammatic analysis of frequency and displacement fields, utilising observations for Bell Rock for the three months September, October and November 1951-55 - the sample being composed of mean hourly wind speeds and directions for the 6o minutes ending 0400 and 1600GMT. Characteristic features of the wind field were noted.

8. The attempt to deduce typical consequences of synoptic influences, and others tentatively associated with topographical features (and regarded as mechanical constraints), is approached through the examination of "seasonal" observations from four stations, viz. Ocean Weather Ship "Juliet", "Northice" (North Greenland), Bell Rock and

Lerwick Observatory, representing respectively a freely exposed station, a land station on an extensive level surface, an off-shore station, and a well exposed land-based station. For the marine winds the period analysed was 1950-55, for the Arctic Station, November 1952 to June 1954, and for the other two, 1951-1955.

The accuracy and adequacy of the several observational samples have been examined in detail and reasons advanced to justify regarding them as representative. Inter alia much data has been processed for the first time and useful information obtained on the statistical value of sub-samples.

9. Detailed graphical analyses of the data, on a seasonal basis, have been carried out, special attention having been paid to the partitioning of frequency and displacement by direction - twelve 30^{o} sectors were used.

The main conclusions may be stated as follows:

(a) For an ideally exposed station sited in the general region of the British Isles.

(i) In "summer", "autumn" and "winter" the distribution of frequency (F) and displacement (D) by direction is uni-modal, the vector mean lying in the sector of highest (D) and (F). A finer analysis shows local concentrations of both (F) and (D) embedded in the frequency density field, mainly in a sector to the south and one somewhat north of west. In "spring" there is evidence that a greater variety of pressure systems operate than in the other seasons, mod is small, and a multi-modal pattern of (F) and (D) results.

(ii) On a seasonal basis $40\% < q < 60\%$, except for "spring" when $q < 20\%$. Average values of q for certain months of given name may exceed 60%.

The actual value of $\sigma/|\bar{V}|$ for a particular season or month of given name, agrees to two-figure accuracy with the value appropriate to a normal circular distribution of parameter q, (such values we may conveniently term the "theoretical" or "expected" values).

$S/\bar{V}_{(S)}$ varies appreciably, however, but is rarely greater than the "expected" value.

(b) For well-exposed stations on or near the British Isles.

(i) The seasonal pattern of (F) and (D) by direction is definitely bi- or multi-modal, certain preferred directions appearing persistently and roughly independently of season. Such "anomalies" are considered to be a characteristic of the station and hence related to its topographical setting. These anomalies are more clearly delineated, and not initially revealed, by comparison of the actual pattern with that appropriate to the normal circular distribution of parameter q.

(ii) The seasonal value of q lies within 25% to 45% except for "spring" when it is less than 20%. A value exceeding 50% should, it is suggested, be considered high.

For the parameter $\sigma/|\bar{V}|$ there is an approach to two-figure agreement between the actual and "theoretical" values.

$S/\bar{V}_{(S)}$ deviates considerably from the "expected" value, which, since $q \leqslant 50\%$, is constant at a figure slightly exceeding 0.52. There is a definite tendency for $S/\bar{V}_{(S)}$ to exceed the "theoretical" value, and this is partly ascribed to a relatively high proportion of calms.

(c) Analysis of the winds at "Northice" - where it seems clear that the air-flow is dominated by a katabatic wind - shows that a distribution for surface winds can occur with a very high value of q viz. 90% or more, implying close concentration about the vector mean.

10. An additional method has been developed for the ready calculation of the angle of incidence θ of the direct solar beam on to a plane of any slope and aspect.

The choice of the coordinate system emphasises certain geometrical features of the problem, and allows the angle θ to be expressed in the form

$$\cos\theta = a + b\cos(t+\varepsilon)$$

where a, b and ε are functions of the geometry of the surface, the latitude and the solar declination, and t the hour angle with respect to a horizontal plane at the point in question. A graphical solution to the above equation permits θ to be determined for any t, given the values for any two selected points.

11. To render a more satisfactory service to industry and agriculture, and to meet more adequately known requirements, consideration might be given to further work on the following lines:

(1) A progressive codification of information on the relations between air flow and surface "roughness" of different scales, in which task the definition of "roughness" might be clarified by the use of certain theoretical models, amongst them, systems of simple barriers, cropped areas such as orchards (i.e. such as present a uniform distribution of closely similar elements), and uniform forest stands. The provisional extension to landscape features generally might be assisted by some form of numerical description of ground contour.

Confirmation of conclusions, based on many scattered sources, regarding the interaction between the degree of slope and winds of moderate strength or greater.

(2) The graphical and statistical methods dealt with in Part II might be applied more generally to observations from a large number of anemograph stations to establish "norms" of wind behaviour in relation to topographic and other influences. Such an investigation might be assisted by a study of theoretical models other than the normal circular distribution: such studies could result in an increasingly quantitative evaluation of the effect on air flow of topographical features on all scales.

These techniques, if successful, could then be applied to data from stations supplying a less complete sample of observations, (in the limit, from stations which may not report as frequently as once daily).

Methods analysing the near-simultaneous observations of surface and upper winds (at heights $<$ 1000 ft.) from stations suitably equipped, and the sequential behaviour of wind-speed and direction at a point by the correlation coefficients of "simple stretch" and "turn", might be employed to reveal characteristic differences between winds at a completely freely exposed station and at an ordinary land-based station.

(3) Conclusions reached on the topics mentioned in the above sections should materially aid rational decisions regarding anemograph net-works, types of instruments, methods of data reduction, etc., appropriate to the tolerances permissible in any given field of application.

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A P P E N D I C E S

Appendix I (a)
The Numerical Expression of Ground Contour

The importance of the specification of surface geometry on all scales in meteorological studies has been stressed in the body of the work, parameters of use in micrometeorology have been noted and some, mainly qualitative, suggestions have been made in relation to larger scale phenomena. In addition, there are the idealised profiles of hills and mountain ranges adopted in investigations of lee waves and related phenomena, and the various attempts to describe the characteristics of the "exposure" of a climatological station. Geographers and cartographers have also proposed empirical procedures for expressing the geometrical properties of the earth's surface. Some methods in current use will now be reviewed briefly and a new technique outlined for specifying, mumerically, the degree of departure of some features of the actual surface from the infinite horizontal and level plane of short turf which is the idealised norm for climatological stations.

The scale of the atmospheric motion under consideration will obviously dictate the size of the significant unit, e.g. when dealing with mountain waves differences in altitudes of a few hundred feet are unlikely to be generally significant; whilst on the micrometeorological scale, height differences of a few centimetres can be of importance.

Among the surface characteristics which require numerical expression are:-

(i) height of horizontal surface;

(ii) slope and aspect of plane surface, and "mean" slope and aspect of a rough, i.e. dissected, surface;

(iii) breaks in the plane surface, and the number and the position of such breaks with respect to the origin;

(iv) mean and extreme amplitudes of variations in height from the plane surface;

(v) the number, per unit horizontal length (or area), of the "oscillations" of altitude about some mean position.

In alternative terms we may postulate that the chosen parameter(s) should respond to the number of turning points, number of contour crossings; number, position with respect to the origin, extent of and degree of ascending (descending) sequences; the mean slope, the total length of track (in vertical cross section) or total area and the range of variation of altitude along any section, or within unit area.

Some of the methods for the evaluation of "mean slope" are conveniently listed in a text by Monkhouse and Wilkinson (1952) and a brief review is given below.

Another approach to the problem is through the adaptation of well established statistical techniques used for the analysis of time-series and for industrial quality control. In these fields of application, a common feature is attention to the order in which ranked or numerical varieties are generated. An obvious starting point is the examination of the permutations of n distinct quantities; leading to a consideration of the order of sign changes, inequalities, etc. and of ascending and descending sequences: the special case where the n quantities may be grouped into two non-overlapping classes is dealt with by analysing the distribution of "runs", etc. If n (not necessarily all different) variates are generated at equal intervals of time, or are equally spaced along a single linear dimension a type of "time series" analysis then becomes appropriate. Until such methods have been used successfully to deal with a few typical climatological problems, an exhaustive analysis of their potentialities is premature. A convenient first reference for meteorologists is a review article by Sneyers (1955), and a useful source a standard statistical text by Hoel (1947). All that will be done in the present study is to outline methods of measuring mean slope and to sketch an approach, believed to be new, to the question of "topographic roughness".

Section 1. The "mean slope" of ground surface

One of the earliest attempts to evaluate an angle of slope appears to be that of Finsterwalder (1890) who defined an angle $\bar{\beta}$ by:

$$\tan \bar{\beta} = \frac{\text{total length of contour x contour interval}}{\text{horizontal projection of area}}$$

In Fig.I(a)(1) let (a b c d) be a small sloping plane area, (ab) and (cd) two successive contours and (a'b'c'd') and (a"b"c"d") respectively the projections of (a b c d) on the vertical and horizontal planes.

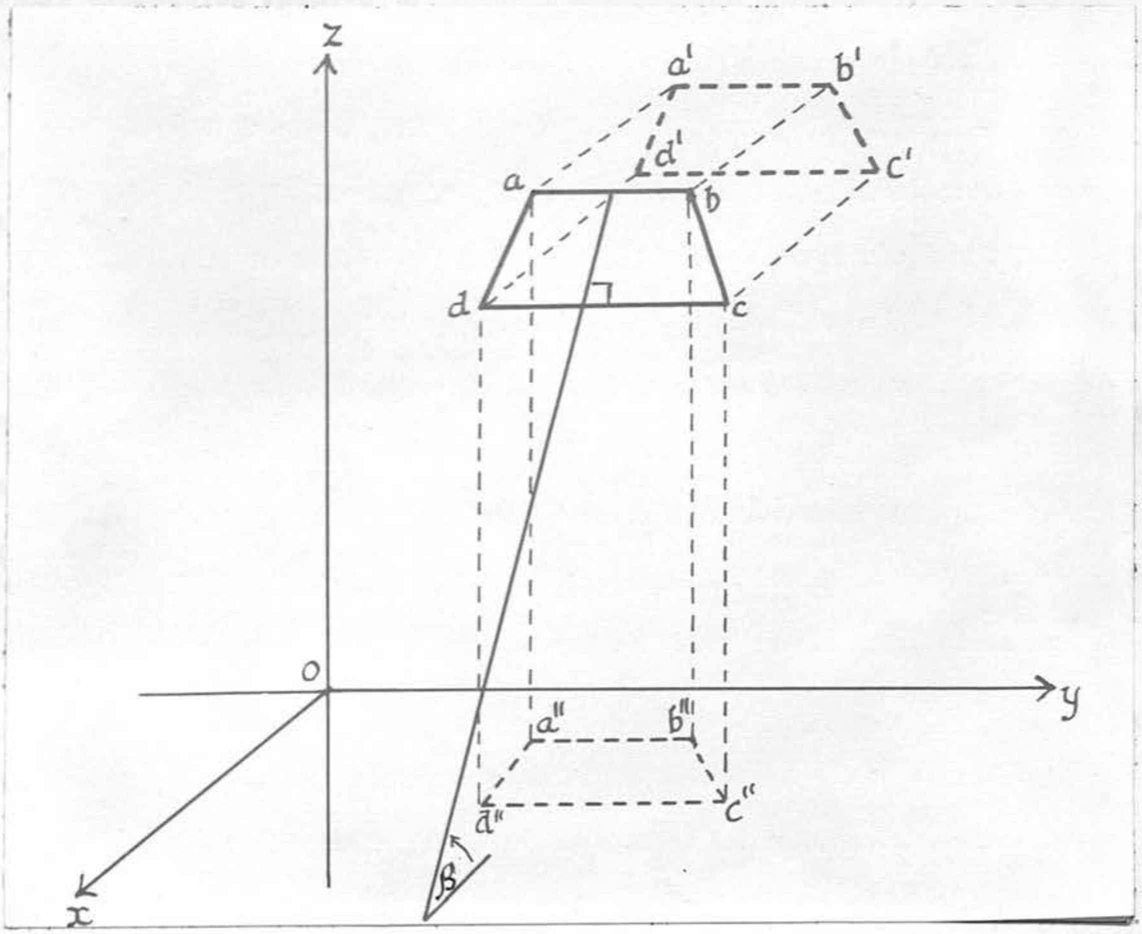

Figure I(a)(1). To illustrate the derivation of the mean angle of slope (β), of a plane surface (a b c d).

Defining the slope of (a b c d) by $\tan\beta$, we have

$$\tan\beta = \frac{(a\ b\ c\ d)\sin\beta}{(a\ b\ c\ d)\cos\beta}$$

But $(a\ b\ c\ d)\sin\beta \simeq (cd \times (bc).\ \ \sin\beta \quad (\equiv (cd) \times I)$

$$\simeq l \times I$$

where l = length of portion of one of the contours (e.g. (cd))

I = contour interval

Summing over all areas we have

$$\tan\bar{\beta} \simeq \frac{I \sum l}{\sum (a''b''c''d'')}$$

$$\simeq \frac{\text{total length of contour} \times \text{contour interval}}{\sum(\text{horizontal projection of elementary areas})}$$

If a single vertical section is taken through the surface then the general expression reduces to

$$\tan\bar{\beta} \simeq \frac{\text{number of contour crossings} \times \text{contour interval}}{\text{horizontal length of transect}}$$

Rich (1916) suggested that a section orthogonal to the contours would suffice, but Wentworth (1931) found such a procedure to lead towards valleys and away from peaks, and aimed to remove this restriction by counting contour crossings per mile length along north-south and east-west lines and repeating this procedure with oblique grids. This second stage involves the argument that the mean number of contour lines ($\bar{N}$) crossed by n unit transects making random angles with the contours, is given by

$$\bar{N} = \frac{\frac{1}{n}\sum_{0}^{\pi/2} \text{number of crossings}}{\frac{1}{n}\sum_{0}^{\pi/2} \sin\theta}$$

and since the mean value of $\sin\theta$ from 0 to $\pi/2$ is $2/\pi$ (= .6366) the result may be expressed in the form

$$\tan\bar{\beta} = \frac{\text{Average number of crossings per mile} \times \text{contour interval in feet}}{5280 \times .6366}$$

The method gives reproducible results if the total number of contour crossings along a transect is of the order of 100 and applies most satisfyingly to areas "not unduly characterised by one way slopes or large valleys".

Raisz (1948) has advocated a "coefficient of land slope" in which various slope categories are weighted by the area under a particular category.

The obvious need to discriminate between an uninterrupted rise or fall of the land and oscillations of the surface led to methods typified by those of Huggins (1935) who measured the extreme height differences in small areas and charted the subsequent distribution of amplitudes. On this topic, however, much can be inferred from a value of "mean slope" obtained by families of parallel transects in two perpendicular directions, as will be immediately evident from a consideration of "mean slopes" derived from simple solid figures.

Some idea of the types of numerical result which arise may be gleaned from some unpublished work by Thomas (1957) which the writer has been granted permission to quote. Thomas adopted the 10 Km. square of the National Grid as the area for study using a north-south, east-west grid. Six areas were examined and contour crossings were counted along the 10 north-south and 10 east-west one kilometre grid lines, and also along the single central north-south and east-west lines. The six chosen areas were deemed to represent six different categories defined in terms of

C = average number of crossings per mile x contour interval in feet

The contour interval was 50 ft. and hence, if the total number of crossings is 130, we have $C = (130 \times 50)/(10 \times \frac{5}{8}) = 1040$. Furthermore, if C is in feet per mile, C = 1000 implies a "mean slope" of 1 in 5.

Table I(a)(1). Expression of "mean slope" of six selected areas
(Thomas, 1957)

Area	Category	Number of contour crossings per 10 km. length					
		Ten Transects			Single Transect		
		E-W(av)	N-S(av)	Mean	E-W	N-S	Mean
Snowdonia	(i) C>1000	124	143	133	125	150	138
S. Wales Uplands	(ii) 750<C<1000	93	96	94	79	99	89
Forest of Bowland	(iii) 500<C<750	71	66	68	80	72	76
Hinterland of Aberystwyth	(iv) 250<C<500	52	68	60	55	65	60
N.E. Shropshire	(v) 100<C<250	15	13	14	18	15	17
Vale of White Horse	(vi) <100	6	6	6	7	4	5

Equality between results for an east-west and a north-south transect obviously imply some symmetry in the form of the surface, but it is not the symmetry of a single feature such as a centrally placed conical hill, if at the same time there is broad equality between the central N-S (E-W) transect and the associated ten transects. Equality in the number of contour crossings - on the one hand as between east-west and north-south transects, and simultaneously on the other hand as between different parallel transects in some other direction - must imply that the actual surface displays a large number of separate, randomly distributed, "humps" whose linear dimensions are of the order of miles and not tens of miles. The self-consistency of various estimates indicate that the results are not fortuitious, but without a background of case histories it is not possible to judge whether the differences are of practical significance or not. It is possible to visualise a single ridge with an axis running

NW-SE or SW-NE giving equality in the numbers of contour crossings in a N-S and E-W direction obtained from multiple and from single transects, but such a situation is hardly likely to be encountered in practice. The results for Snowdonia and the Aberystwyth hinterland suggest that the main ridge lines would run east-west rather than north-south, and north-south rather than east-west for the Forest of Bowland. In a later communication Thomas (1958), from a study of the map, kindly supplied rough subjective estimates of the direction of the main ridge for the six areas; his conclusions were as follows:-

(i) NW to SEE (ii) N to S (iii) N to S (?)

(iv) WNW to ESE (v) NE to SW (vi) E to W

with which the writer's suggestions are concordant.

The "mean slope" of ground surfaces in non-mountainous terrain is unlikely to exceed about 1 in 5: and it follows that the additional area of contact between the atmosphere and the surface in hilly country as compared with horizontal terrain is unlikely to exceed a few percent.

In this approach, as in any which involve mapping techniques, questions of scale assume fundamental importance, and only experience with the technique can provide the necessary expertise.

A further development is set out in the next section, stimulated by the need to differentiate between surfaces characterised by extensive uninterrupted slopes, and those in which an equal number of contour crossings could be generated by oscillations about a mean level.

Section 2. Outline of a possible new approach to the numerical specification of ground contour

Techniques are sought which will give numerical expression to such ideas as slope and "ruggedness". Whilst a numerical measure permits a better comparison between cases of the same general class, the physical significance of the property measured depends upon other considerations and this contribution is merely concerned with the development of a method which may be relevant to certain climatological problems. The treatment is limited to the two-dimensional case.

Suppose (Fig.I(a)(2)) Ox and Oz be horizontal and vertical axes from an arbitrary origin O at (say) mean sea level, and let the (m + 1) points be P_0^1, P_1^1 ..... P_M^1

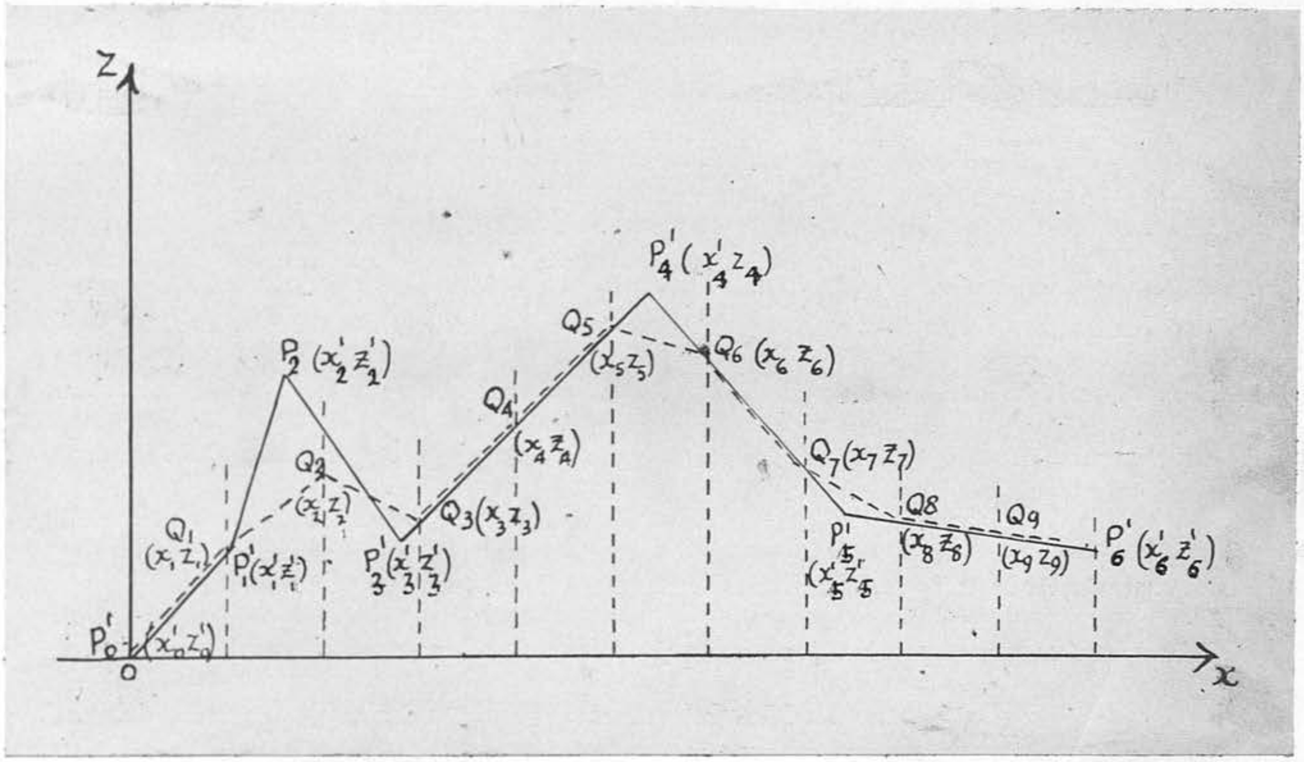

Figure I(a)(2). To illustrate:
an actual cross-sectional profile
$P_0^1 P_1^1$...
and a modified cross-sectional profile
QoQ1 ..

If p be the contour interval then $\sum |z'_r - z'_{r-1}|/p$ gives the number of contour crossings. Suppose $l_1 = P'_0 P'_1$ $l'_m = P'_{m-1} P'_m$ then $\sum_1^m l'_r$ will be the perimeter of the profile and

$$l_r' = \left[(z_r' - z_{r-1}')^2 + (x_r' - x_{r-1}')^2\right]^{\frac{1}{2}}$$

$$= (x_r' - x_{r-1}')\left[1 + \left(\frac{z_r - z_{r-1}'}{x_r' - x_{r-1}'}\right)^2\right]^{\frac{1}{2}}$$

hence $$\sum l_r' = \sum\left\{(x_r' - x_{r-1}')\left[1 + \frac{1}{2}\left(\frac{z_r' - z_{r-1}}{x_r' - x_{r-1}'}\right)^2\right]^{\frac{1}{2}}\right\}$$

if $(z_r' - z_{r-1}')/(x_r' - x_{r-1}')$ is small enough for powers greater than its square to be neglected.

Now consider a "modified" profile in which the horizontal intervals ($x_r - x_{r-1}$) are equal. Some information will be lost, e.g. the "peaks" and "troughs" of the actual profile will not necessarily occur with the selected value of x, however the loss can be reduced by a change of scale or by using a series of interpenetrating samples.

Statistical tests may be used to determine how far the number of ascending or descending sequences deviates from expectation. Amongst suitable tests which do not depend upon the absolute value of the variate z are the following:-

(i) Yule and Kendall (1950) in respect of the number of turning points.

(ii) Wald and Wolfowitz (1940), and Swed and Eisenhart (1943) in respect of runs of alternatives (the alternative in this case would be the signs of ($Z_r - Z_{r+1}$)

(iii) Mann, H. (1945), who developed a test against linear trend based on the number of inequalities in a sequence of distinct quantities.

These tests, and others to be considered, depend essentially on the hypothesis that a random permutation of a sequence of distinct quantities will exhibit neither too many "runs" nor too many "oscillations". The sensitivity of such tests to the sampling technique adopted is immediately obvious when we note that, with the type of profile illustrated in Fig. I(a)(2) – when $Q_r Q_{r+1}$ is a straight line – the number of runs or sequences may be indefinitely increased by increasing the frequency of sampling. If, however, our method of measuring z is such that it responds to those small variations between adjacent values of z which are obscured by the nominal straight line, a more intensive sampling might be expected to reveal oscillations as readily as it reveals runs.

Appropriate statistical techniques for numerical variates, based upon serial correlation might be developed on the following lines.

Denoting the horizontal interval by d and successive values of z in a particular permutation of the z family by z_o, z_1, z_n then,

$$l_r^2 = (z_r - z_{r-1})^2 + d^2$$

$$l_r = d\left[1 + \left(\frac{z_r - z_{r-1}}{d}\right)^2\right]^{\frac{1}{2}} \simeq \left[1 + \frac{1}{2}\left(\frac{z_r - z_{r-1}}{d}\right)^2\right]$$

since $(z_r - z_{r-1})/d$ will almost invariably be small.

Hence $$\Sigma l_r = nd + \frac{1}{2d}\left[\Sigma z_r^2 + \Sigma z_{r-1}^2 - 2\Sigma z_r z_{r-1}\right]$$

and Σl_r will vary from one permutation to another mainly by virtue of changes in $\Sigma z_r z_{r-1}$, although a secondary source of variation is introduced by different values of z taking the terminal positions. Neglecting this latter source of variations and assuming $z_0 \simeq z_n$ we have

$$\sum_1^n l_r = nd + \frac{1}{2d}\left(2\Sigma z_r^2 - 2\Sigma z_r z_{r-1}\right) \qquad \text{I(a)(1)}$$

and hence approximately

$$\text{Perimeter} = \text{Constant} - \frac{1}{d}\Sigma z_r z_{r-1}$$

for a particular family of z's.

For a given family of "modified" heights the perimeter is a minimum when $\sum z_r z_{r-1}$ is a maximum (i.e. when the z's are in ascending or descending sequence), and takes a maximum value when large and small values of z are juxtaposed. These two types of arrangement correspond to a smooth and to a rugged profile.

Wald and Wolfowitz (1943) have investigated the properties of the important term

$$(R \equiv) \quad \sum z_\alpha z_{\alpha+h}$$

in the expression for the serial correlation coefficient R(h) of lag h, viz:

$$R(h) = \frac{\sum_{\alpha=1}^{N} z_\alpha z_{\alpha+h} - \left(\sum_{1}^{N} z_\alpha\right)^2 / N}{\sum_{\alpha=1}^{N} z_\alpha^2 - \left(\sum_{1}^{N} z_\alpha\right)^2 / N}$$

Using a circular statistic (the necessary modification for a non-circular statistic has been given by Watson and Durbin (1951)), they show that the expected value E(R) is

$$E(R) = \frac{1}{N-1}\left(S_1^2 - S_2\right)$$

where $S_K = z_1^k + z_2^k \cdots + z_n^k$, and that $\sigma^2(R)$ may be expressed in terms of the sums of powers of z'. They further show that, providing N is prime to h, the distribution of $\sum_{\alpha=1}^{N} z_\alpha z_{\alpha+h}$ is exactly the same as that of $\sum_{\alpha=1}^{N} z_\alpha z_{\alpha+1}$.

Certain profiles were examined using R(h), amongst them that for a north-south transect from Queensferry, Firth of Forth, to the upper Tweed valley near Broughton, Peebleshire (30 miles).

Heights in hundreds of feet were noted at mile intervals and values R(1) and R(2) for lags 1 and 2 were computed for:

(i) the actual "modified" profile;

(ii) a profile in which vertical ordinates arrange in ascending order;

(iii) a profile with the highest value in the middle;

(iv) a rearrangement corresponding to a subjective estimate of a profile with maximum roughness.

Transect	Contour Crossings	R(1)	R(2)
(i)	51	2827	2648
(ii)	38	2085	2654
(iii)	37	2954	2885
(iv)	278	1482	2930

It will be noted that the highest value for R(2) is that for transect (iv) which, in its basic form, is the most rugged of the profiles, being constructed by associating large and small values to give a serrated edge. The computation of R(2) however requires the products of alternate ordinates, and hence the association of values of ascending magnitude.

Other results are as follows:

$$E(R) = 2188$$

Mean slope (after Thomas, 1957), 1/32 for transect (i) - in this case 13 of the 30 stages are of greater slope, and 17 of less slope than the mean value. For transect (ii) (ignoring the sharp drop from the last point to the repeated value of the first) mean slope is about 1/75.

The expected number E(T) of turning points for a series of 31 observations is 19, for Transect (i) T = 13 and $\sigma(T) = 2.3$.

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Appendix I (b)

Some Comments on the Siting of Anemometers

The correct siting of anemometers is very dependent upon an appreciation of the phenomena discussed in this thesis, and it may be useful to set out some facts for consideration.

Comprehensive data on "surface" wind speed and direction are required for:-

(i) Synoptic and macro-climatological purposes: for which purpose the ideal site would be one on an infinite level smooth turf surface.

(ii) For the purposes of local forecasting and to provide data for industrial, agricultural and similar needs: in which case the recorded data should reflect the major local influences (e.g. topography, sea breezes).

(iii) For micro-meteorological and similar purposes where a precise knowledge of the airflow characteristics at a given place are required (e.g. diffusion of pollution and radioactive material, ventilation and heat loss of individual buildings or groups of buildings).

Frith (1956) in a memorandum to the Meteorological Research Committee considered that the W.M.O. definition of surface wind, viz. that at 10 metres above ground in an open situation, would imply that "it would be unreasonable to attempt to make the measurement more 'representative' by removing from it any component due, for example, to local topography or to a sez-breeze"; thus, however desirable an open network of perfectly exposed anemometers might be in practice, data of the type covered by (ii) will be the most that can be expected in the vast majority of instances. For the purposes outlined in (iii) only a sufficiency of on-site instruments will normally be adequate.

As a tentative generalisation, it is suggested that an anemometer should not be in the turbulent wake of an individual upwind obstruction, but nevertheless should be within the fully developed turbulent wake of the assemblage of upwind obstructions, and the following suggestions are advanced in conformity with this general approach.

The mechanical agencies disturbing airflow may be classified as:-

(i) Topographical features such as undulating ground, of a type which does not give rise to a "break-away" of the surface boundary layer.

(ii) Topographic features which give rise to such a "break-away".

(iii) Topographical features leading to local confluence and/or deflection of airflow.

(iv) Irregular vegetation, outcrops, boulders and artificial obstructions, each of which gives rise to eddying motions recognisable on a scale of feet.

For lapse rates other than adiabatic, provisional simplifying assumptions might be as follows:-

(i) With large lapse rates wind speed will either be very light, or will be sufficiently strong to render the departure from a logarithmic vertical profile of minor significance.

(ii) In stable conditions there will be a tendency to either light winds, or to a laminar flow, in which case the influence of upwind roughness tends to be damped, and over undulating terrain, breakaway of the boundary layer will be inhibited.

which?

The characteristic of a stable flow which would seem to be of most relevance is that the boundary layer associated with a bluff obstruction retains its form for a greater downwind distance than in a more turbulent stream and the "wind shadow" is more extensive. But in this case, as with other well-marked phenomena occurring with stable conditions (e.g. katabatic winds, "helm" winds, mountain-valley circulations), other characteristics associated with the flow permit the recognition of these phenomena and the problem is more properly one relating to the analysis of observations rather than to siting.

Some general proposals

1. Mainly in respect of the evaluation of new sites

1.1. Over undulating country with a surface of short turf, an anemometer head at 10 metres from the surface may be accepted, providing there is an uninterrupted fetch in all directions of 30H or more from a hill of height H.

1.2. Cases will frequently occur where it is not possible to place an instrument as indicated above, and it will be necessary to site it on rising or falling ground, on the shoulder or summit of a hill, etc.

If the ground slope is less than about 15°, the 30H condition may be relaxed. Over ridges and summits there will usually be some compression of the streamlines, and it will be generally impracticable to place the anemo-head sufficiently high to be in the undisturbed flow.

It may be added that the air stream will move smoothly up slopes of less than about 30° to the horizontal.

1.3. In hilly areas it may often be impossible to find a site obeying 1.1., but one can be found where the air flow is deflected rather than obstructed by the lie of the land. In these circumstances a reliable sample of the deflected flow may be obtained with an anemo-head at 30 ft.

A general wind perpendicular to the line of a valley will, however, lead to very irregular record; but this will be a consequence of "correct" siting with respect to the deflected wind, as the steady wind along the valley may necessarily be associated with an eddying flow across the valley.

1.4. In the presence of individual obstacles of height H the anemo-head should be 3H from the ground within the vicinity of the obstacle.

This particularly applies to cases where an anemometer is to be placed within the confines of a group of buildings of general height H.

1.5. If placed 30H downwind from individual bluff obstacles, it is postulated that under adiabatic conditions a logarithmic profile will have been re-established.

1.6. In cases of an extended rough surface in which the individual elements are of approximately the same form and height H (e.g. a forest, a suburban housing estate, an extensively cultivated area), a tentative suggestion is that the anemometer head should be at least 2H above the ground.

Some observations above an extended rough surface - that formed by the tops of trees in a forest, have been made, Fons (1940). The height at which the wind speed over the canopy equalled that which would have been attained had the canopy been absent increased with the increase of absolute wind speed, e.g. at 5 m.p.h. equality was reached at a small fraction of H above the canopy; at 15 m.p.h. equality was not reached at a height much exceeding H from the top of the canopy. (H = height ofcanopy).

1.7. In cases where there is a marked change in surface roughness, e.g. as when open agricultural land gives way to a built-up area, it is desirable to ensure that the boundary layer appropriate to the new surface has been established. Sutton (1953) suggests that when the height of the new roughness elements is only of the order of tens of centimetres, the ratio of anemo-height to distance from the leading edge should be at most 1:10; Deacon, however, (1953) advises a much smaller ratio, and the former author states that if the obstacles are much larger (e.g. houses) the above ratio must be much reduced.

2. <u>In respect of existing instruments</u>

One may postulate that an instrument is unsatisfactorily exposed if:-

2.1. The appropriate conditions under 1.1. are not satisfied.

2.2. Under adiabatic conditions the logarithmic profile is not obtained.

2.3. The speed momentarily falls to zero in moderate or strong winds (a consequence of back eddying).

2.4. The direction trace shows signs of complete rotation or too frequent oscillation through 180°.

2.5. Mean speeds evaluated over 10 mins. and over groups of 15 sec. intervals do not exhibit a satisfactory relationship (this latter to be defined). The divergence between means in, say, adiabatic conditions, might be regarded as a function of the inadequacy of the exposure. A comparison of means over 15 sec. and 10 mins. has, it is understood, been undertaken at Prestwick Airport, but work by Best and Scrase on gustiness with bi-vanes suggests that 15 sec. is far too short an exposure for a representative sample of flow; Best's exposure time was 1 min. However, these are matters for study.

2.6. "Gustiness" is not proportional to mean speed, and "gustiness" as judged from wind direction is unacceptably related to that obtained from wind speed.

3. Only experience can decide how much attention it is desirable or practicable to devote to the techniques implied in the above analysis.

What, however, does seem clear is that the background of knowledge of the possible effects of obstructions on air flow is now approaching a stage which might render a re-examination of the problem of siting a profitable enterprise.

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Appendix II (a)

A Tabulated Function for Wind Analysis

Knighting (1954) has pointed out that the cumulative frequency of wind vectors for speeds from 0 to V summed over all direction is given by

$$F(V) = \frac{2}{\sigma_0^2}\int_0^V \exp\left[-\frac{V^2+|\bar{V}|^2}{\sigma_0^2}\right] I_0\left(\frac{2V.|\bar{V}|}{\sigma_0^2}\right) V.dV \quad \text{...........II(a)(1)}$$

σ_0 - the standard vector deviation and $|\bar{V}|$ the vector mean speed.

Whilst working on wind distributions, a publication entitled "Offset Circle Probabilities" (The Rand Corporation) came to my notice, in which an alternative form of Eq.II(a)(1) has been tabulated; a brief account of the contents might be of interest to meteorologists.

Let A be the centre of a normal circular distribution and O the origin of co-ordinates and $\sigma_x (= \sigma_y \equiv \sigma_0/\surd 2)$ the linear standard deviation.

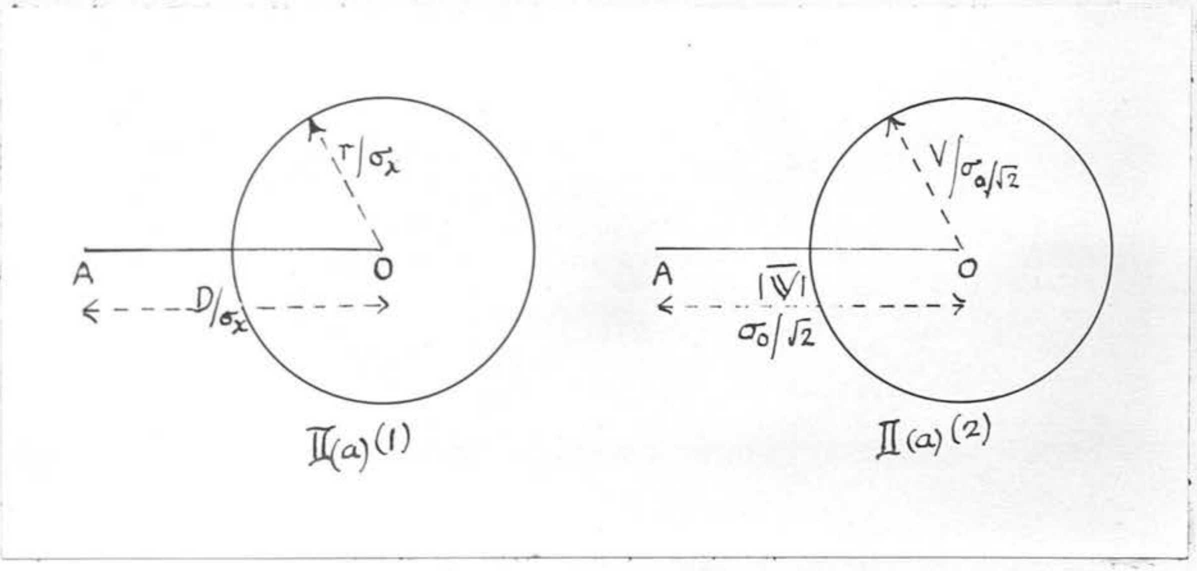

Figure II(a)(1). Notation used in tabulated function (Rand Corporation)

II(a)(2). Notation customarily used in wind analyses (Brooks et al., 1950)

The tables in the publication are set out in terms of

$$(r-D)/\sigma_x = -3.9\ (0.1)\ 4.0 \text{ as abscissa}$$

$$D/\sigma_x = 0(0.1)\ 5.9\ (0.5)\ 10.0\ (1.0)\ 20.0 \text{ as ordinate}$$

and give the probability (q) of a vector end falling <u>outside</u> the region bounded by the circle (0, r/σ_x).

<u>Examples</u> (i) Let $|\bar{V}| = 1$, $V = 4|\bar{V}|$ and $|\bar{V}|/\sigma_0 = 1/2.05$

$$\left[(r-D)/\sigma_x \equiv\right] \frac{\sqrt{2}.V - \sqrt{2}.|\bar{V}|}{\sigma_0} = \frac{\sqrt{2}.3.|\bar{V}|}{\sigma_0} = 2.079$$

$$\left[D/\sigma_x \equiv\right] \frac{\sqrt{2}|\bar{V}|}{\sigma_0} = \frac{\sqrt{2}}{2.05} = .690$$

giving q = .043

or p (= 1 - q) = .957 (compared with .958 from GM 85 , Table XXVIII).

(ii) $|\bar{V}| = 8.8$ knots. $V = 21.1$ knots $(\equiv 2.4|\bar{V}|)$

and $\bar{V}(s)$ (mean scalar speed) = 22.0 knots

Hence $|\bar{V}|/\bar{V}(s)$ = 40% corresponding to $\sigma/|\bar{V}|$ = 2.64 (GM 85, Table XXIX).

Thus

$$\left[(r-D)/\sigma_x \equiv\right] \frac{\sqrt{2}(V-|\bar{V}|)}{\sigma_0} = \frac{\sqrt{2}.(1.4)}{2.64} = 0.750$$

$$\left[D/\sigma_x \equiv\right] \frac{\sqrt{2}.|\bar{V}|}{\sigma_0} = \frac{\sqrt{2}.(1)}{2.64} = 0.536$$

giving q = .487 hence p = .513 compared with .514(6) from GM 85, Table XXIX.

The "Rand" tables extend to cases corresponding to high values of $|\bar{V}|/\bar{V}(s)$, and in general a double interpolation is required.

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Appendix II (b)

On Some Computational Procedures

When computing the statistical parameters with respect to speed or direction or both, grouping of observations is almost unavoidable. In many cases the basic data will be grouped (e.g. Beaufort force categories for maine data), and in practically all cases the arithmetic becomes unmanageable without this device, and it is therefore necessary to examine in some detail the type and extent of errors involved in the procedures. In particular the need for a type of correction advocated by Brooks et al. (1953, p. 196), arising from the grouping process will be considered (see Section 2).

Section 1. Errors in mean speeds

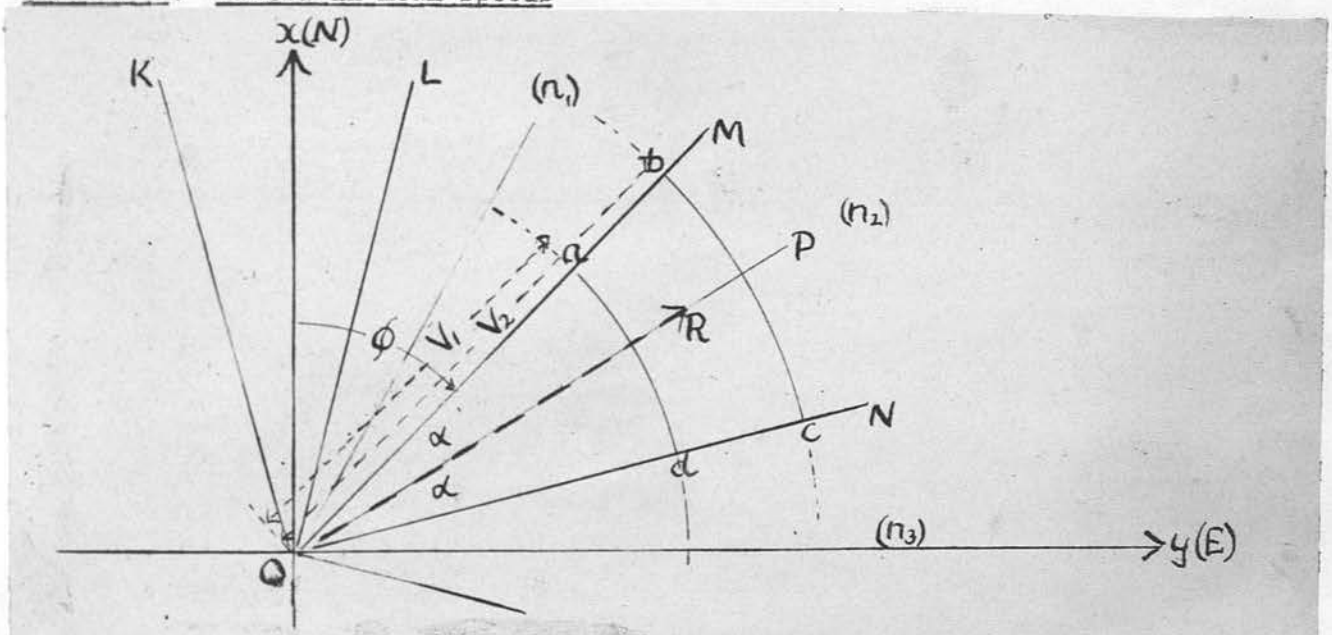

Figure II(b)(1). Schematic diagram for examination of grouping corrections.

Suppose the two-dimensional vector field be divided into areas such as (a b c d).

Initially all vector ends occurring within the area are (by convention) regarded as being concentrated at a point R lying at distance

$(V_1 + V_2)/2$ along the bisector OP of the angle MON ($= 2\alpha$). If it is reasonable to assume that the frequency density of vector ends within (a b c d) is uniform, then the position of the mean centre (R') may be derived as follows:

Obc and Oad are sectors of a uniform lamina of which the centres of mass will lie at distances

$$\frac{2}{3} V_2 \frac{\sin\alpha}{\alpha} \quad \text{and} \quad \frac{2}{3} V_1 \frac{\sin\alpha}{\alpha}$$

where $\quad V_2 = Ob. \quad$ and $\quad V_1 = Oa$

The "mass" of the portion (a b c d) may be written

$$\left(V_2^2 - V_1^2\right)\alpha$$

and if R' be the true position of the mass centre

$$\left(V_2^2 - V_1^2\right)\alpha . OR' = \frac{2}{3}V_2 \frac{\sin\alpha}{\alpha} V_2^2\alpha - \frac{2}{3}V_1 \frac{\sin\alpha}{\alpha} . V_1^2\alpha$$

$$= \frac{2}{3} \sin\alpha \left(V_2^3 - V_1^3\right)$$

$$\text{hence} \quad = \frac{4}{3} \frac{\sin\alpha}{\alpha} \left[\left(\frac{V_1+V_2}{2}\right) \frac{V_1^2 + V_1V_2 + V_2^2}{(V_1+V_2)^2}\right]$$

$$= \frac{4}{3} \frac{\sin\alpha}{\alpha} (OR) \left[1 - \frac{V_1V_2}{(V_1+V_2)^2}\right]$$

The error is least as

$$\frac{4}{3} \frac{\sin\alpha}{\alpha} \left[1 - \frac{V_1V_2}{(V_1+V_2)^2}\right] \longrightarrow 1,$$

and $\frac{4}{3} \frac{\sin\alpha}{\alpha}$ increases to $\frac{4}{3}$ as α decreases, whilst $\left[1 - \frac{V_1V_2}{(V_1+V_2)^2}\right]$ decreases to (1 - 1/4) or $\frac{3}{4}$ as the absolute magnitude of V_1 V_2 increases and as $V_2 - V_1 = \delta$ (the class interval) decreases.

In the present analyses the class interval is of the order of 6 knots or 3 meters per second, and directional interval 30°. The value of the factor

$$\frac{4}{3} \frac{\sin\alpha}{\alpha} \left[1 - \frac{V_1V_2}{(V_1+V_2)^2}\right]$$

in the successive intervals is:-

(knots)

0-6	6-12	12-18	18-24	24-30	30-36
1.318	1.025	1.002	0.995	0.995	0.993

and 0.989 for all intervals beyond 36 knots. OR thus differs little from the OR (the "true" value) except in the first group, but in the analyses the first class interval has, in fact, been sub-divided when the data so permitted and this materially reduces the rather large proportional error in the first class. 2

Three figure accuracy is barely justified, but it is clear that if a uniform distribution within sections may be assumed, an analysis employing speed intervals of 6 knots and directional intervals of 30 degrees involves errors which, in view of the quality of even the best observations, may be safely regarded as negligible.

Section 2. Errors in mean direction

It is a matter of experience that vector ends tend to cluster around one or more centres, and in the limiting case of the normal circular distribution there would be one such centre. Brooks et al. (1953) suggest that the consequent non-uniformity of point density within the elementary area can be allowed for by arranging for a rotation of a mid-radius such as O P (Fig.II(b)(1)), towards a point of local concentration. If the total frequency of vector ends embraced within successive sectors is (n_1), (n_2), (n_3) ...., then a mid-radius such as O P should be rotated positively by $\frac{n_3-n_1}{n_2} \times \frac{15}{G}$ degrees (G = number of direction groups), before computing the vector means. These authors suggest that such a grouping correction may be desirable at times even if as many as sixteen directional groups are adopted; certainly with 8 groups the correction seems necessary and the need increases as the wind vectors cluster increasingly around a single direction. The procedure outlined in this section was adopted except for the analysis of the Ocean Weather Ship data, where the basic computations were carried out using the thirty-two point compass.

Section 3. Errors in the computation of mean square deviations

A central feature of the anlysis is the computation of the standard vector deviation σ, whilst in this present study attention is also given to the scalar standard deviation (s).

If $\mathbf{V}$ be a vector wind $|\mathbf{V}|$ the module, $|\bar{\mathbf{V}}|$ the module of the vector mean and $\bar{V}_{(s)}$ the scalarmean, then it has been shown that

$$\sigma^2 = \frac{\Sigma |\mathbf{V}|^2}{n} - |\bar{\mathbf{V}}|^2$$

$$s^2 = \frac{\Sigma |\mathbf{V}|^2}{n} - \bar{V}_{(s)}^2$$

The computation proceeds by finding the frequency of wind observations lying within a particular speed range $V_2 - V_1$ irrespective of direction, and multiplying each frequency by a square of the mid-range speed.

Brooks et al. (1953, p.179) however, suggest that this is incorrect for "if we assume that all speeds between V_1 and V_2 are equally likely then the mean square of the speed in the interval V_1 and V_2 is

$$\frac{1}{V_2 - V_1}\int_{V_1}^{V_2} V^2 . dV = \left(\frac{V_1 + V_2}{2}\right)^2 + \frac{(V_1 - V_2)^2}{12}$$

a value which exceeds the square of the mean speed in the interval by one-twelfth the square of the interval. For small intervals this is sufficient"; (these authors infer later that a class interval of 10 knots is a "small" one).

It is necessary to examine the assumption that all speeds between V_1 and V_2 are equally likely and to estimate the extent of the error which the procedure advocated aims to reduce.

Let f(1),f(2)...f(n) be the frequencies, summed over all directions, of wind in class intervals of size δ_1, δ_2 .... and let the mean speeds be assumed equal respectively to the mid-values $\bar{V}_1$, $\bar{V}_2$ $\cdot\cdot$ etc. According to Brooks et al. we should write

$$\Sigma[f_1 V_1'^2] = \Sigma\left[f_1\left(\bar{V}_1^2 + \frac{\delta_1^2}{12}\right)\right] = \Sigma f_1 \bar{V}_1^2 + \Sigma f_1 \frac{\delta_1^2}{12} \qquad \text{II(b)(1)}$$

and hence

$$\sigma'^2 = \frac{\Sigma f_1 \bar{V}_1^2}{n} + \frac{1}{n}\frac{\Sigma f_1 \delta_1^2}{12} - |\bar{V}|^2$$

and similarly for s', where σ' and s' are the "corrected values".

If $\delta_1 = \delta_2 = \delta_3 =$

$$\sigma'^2 = \frac{\Sigma f_1 \bar{V}_1^2}{n} - |\bar{V}|^2 + \frac{\delta^2}{12} = \sigma^2 + \frac{\delta^2}{12}$$

A convenient class interval for wind analyses is about 6 knots ($\simeq$ 6 mph or 3 m/s) and is about the mean interval between successive Beaufort estimates, and since σ is of the order of 10 knots and $\delta^2/12 \simeq 3$ knots the "correction" required to the variance is an additive one of approximately rather less than 3 per cent.

The assumption that, irrespective of direction, all speeds within a given class interval are equally likely was considered in the light of:-

Series (i) 155 observations of surface winds at Bell Rock (0400 GMT readings for the months of January 1951-5).

Series (ii) 471 observations of surface winds at the station "Northice", Greenland (readings at 00h (or 21h) 06h, 12h, 18h; September and October 1953, November 1952, 1953). (Hamilton and Rollitt 1957).

The frequency distributions for intervals of 1 knot are given below for the two series:-

V (knots)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Series (i)	1	4	0	5	0	1	5	6	6	6	5	5	8	5	9	3	5	2
Series (ii)	0	1	0	3	2	8	6	7	11	16	28	28	35	32	32	32	52	34

V (knots)	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
Series (i)	4	2	3	10	5	7	3	5	5	3	5	2	6	1	3	1
Series (ii)	24	22	15	19	17	8	9	6	6	2	7	0	4	0	0	0

V (knots)	34	35	36	37	38	39	40	41	42	43	44	45	46
Series (i)	1	2	2	1	0	1	2	1	0	0	1	1	1
Series (ii)	1	1	2	1	-	-	-	-	-	-	-	-	-

In both cases we note the positive skewness characteristic of many - perhaps the majority - of scalar wind distributions; both series show the property - noted by Brooks et al. (1953, footnote on p.198) - that "wind speeds in the highest ranges observed tend to be near the lower limits (of the class interval)". In series (ii) there is a reasonably good approximation to a unimodal distribution: in series (i) this tendency is evident up to about 15 knots and beyond 21 knots with a deficit of entries in the central portion.

In the subsequent calculations the grouping interval adopted was:-

series (i) 0, 1 - 3, 4 - 6, 7 - 12 and thence by 6 knot steps

series (ii) 0, 1 - 3, 4 - 6 and thence in steps of 3 knots.

Aggregating frequencies into the indicated groups, it is evident from the above frequency analysis that it is incorrect to assume, for these cases at least, that within the groups "all speeds are equally likely".

To examine further the need for this "correction", a comparison was made for the above two series and for three others using a number of procedures, viz.

- A - in which unit frequency intervals were employed: this was assumed to give "the true value", as clearly the smaller the class interval the less cogent the argument for any correction;
- B - in which data are grouped according to the data available into class intervals of 3 knots or 6 knots or Beaufort scale categories;
- C - using grouped data as in B but corrected as suggested by Brooks (see Eq. II(b)(1)).

Methods A, B, C were employed on the series (i); A and B on series (ii) and B and C on:

Series (iii) 744 observations of surface winds at Ocean Weather Ship "J" for all eight synoptic hours in January 1950-1952.

Series (iv) Similar to (iii) but for the month of May 1950-52.

Series (v) The 93 observations provided by the 0400 GMT observation only, Ocean Weather Ship "J" for May, 1950-52.

Observations for the month of May were selected in (iv) and (v) because the relative magnitude of any error will increase as the "uncorrected" variance decreases, and wind speeds and variance are less in the summer than in the winter months.

The results for several parameters are set out below.

Table II(b)(1). Values of Certain Parameters according to Various Procedures using Selected Series of Wind Observations ($|\overline{V}|$, $\overline{V(s)}$, σ and s. in knots)

Series	Parameter	Procedure A	Procedure B	Procedure C
(i) Bell Rock				
155 obs. (0400 GMT)	$\overline{V(s)}$	18.61	18.55	18.55
Jan.1951-55.	s^2	102.58	103.75	106.51
(ii) "Northice"				
471 obs.: four synoptic hrs.				
Sept., Oct. 1953	$\overline{V(s)}$	15.53	15.58	-
Nov.1952, 1953	s^2	31.67	32.53	
(iii) O.W.S."J"	$\overline{V(s)}$		24.01	24.01
744 obs. (8 obs/day)	s^2		104.45	107.27
Jan.1950-52	$s/\overline{V(s)}$	-	0.426	0.431
	σ^2		461.78	464.60
	$\sigma/\lvert\overline{V}\rvert$		1.45	1.46
(iv) O.W.S."J"	$\overline{V(s)}$		15.90	15.90
743 obs. (8 obs/day)	s^2		59.34	61.88
May 1950-52	$s/\overline{V(s)}$	-	0.484	0.495
	σ^2		306.10	308.64
	$\sigma/\lvert\overline{V}\rvert$		7.11	7.14
(v) O.W.S."J"	$\overline{V(s)}$		15.56	15.56
93 obs. (0400 GMT)	s^2		53.39	55.75
May 1950-52	$s/\overline{V(s)}$	-	0.470	0.480
	σ^2		291.74	294.11
	$\sigma/\lvert\overline{V}\rvert$		8.80	8.84

In addition certain other parameters relating to the two-dimensional field of vector ends were computed but the differences following procedures B and C were small and in general less than the errors involved in the graphical methods used in the later analyses.

From the results for series (i) and (ii) it is apparent that the effect of grouping (as given by procedure B) was to increase the value of scalar variance (and necessarily that of the standard vector deviation) above the "true" value (i.e. as given by procedure A). The result of the additional increase as incurred by the procedure advocated by Brooks et al. is illustrated by the comparison of the estimates under the column headed "B" and "C".

It appears therefore, from the evidence presented, that the correction advocated is unjustified, and further that, even if urged on grounds other than those mentioned by Brooks et al., then with the class intervals adopted the magnitude of the difference in parameters is small, a few percent at most, and hardly justifies the extra computational labour, especially in view of error attached to certain graphical and other procedures adopted in the analysis and the quality of the basic data.

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Appendix II (c)

Some Information regarding the Adequacy and Accuracy of the several Observational Samples

Section 1. The Ocean Weather Ship data

In respect of the OWS"J" data, the basic tabulations are available as two series aggregated by months, one series for the three years 1950-52, and the other for the period 1953-55. For each month of given name, the observations extracted were those for:-

A - "all hours" (i.e. the conventional eight synoptic hours);

B - for 0300 GMT;

C - for 1500 GMT.

Other workers have found that wind observations at intervals of less than about 48 hours are not statistically independent. Hence, in the absence of any diurnal or other systematic tendency, statistical parameters derived from the "all hours" series are no more reliable than those derived from the partial series, either singly or in combination. Examination of the data revealed, however, certain advantages in relying most upon the "all hours" series: this was found when constructing frequency density diagrams but was suggested by the following comparison of basic parameters computed from the three series (A), (B), (C).

If series (A) does provide a better sample, then it would be consistent if the values of $\overline{V}_{(s)}$, ϕ, s, $\sigma_{(o)}$, $\sigma_{(N)}$, $\sigma_{(E)}$ and possibly $s/\overline{V}_{(s)}$ derived from (A) tend to be intermediate between those obtained from series (B) and (C). Furthermore, in "GM 85" (p.24) it is pointed out that an unrepresentative sample might well result in a paucity of observations in directions furthest from the resultant direction - this would lead to an inflated value for $|\overline{V}|$, and could also be associated with too low a value for $\sigma_{(o)}/|\overline{V}|$ and too high a value for q.

Table II(c)(1). To show the Relative Magnitudes of the Values of the Basic Parameters of the several "Ocean Weather Ship" series:

"l" - number of occasions (i.e."months") when values from series (A) less than from either (B) or (C);

"m" - number of occasions (i.e."months") when values from series (A) intermediate between those from (B) or (C);

"g" - number of occasions (i.e."months") when series (A) greater than from either (B) or (C).

	Period								
	1950-52			1953-55			1950-55		
Parameter	l	m	g	l	m	g	l	m	g
$\overline{V}_{(S)}$	4	6	2	1	11	0	5	17	2
ϕ	2	7	3	2	8	2	4	15	5
S	1	9	2	1	10	1	2	19	3
$\sigma_{(\theta)}$	2	8	2	0	11	1	2	19	3
$\sigma_{(N)}$	0	12	0	1	10	1	1	22	1
$\sigma_{(E)}$	0	9	3	4	7	1	4	16	4
$S/\overline{V}_{(S)}$	1	9	2	1	9	2	2	18	4
$\lvert\overline{V}\rvert$	4	8	0	7	5	0	11	13	0
q	3	8	1	6	6	0	9	14	1
$\sigma_0/\lvert\overline{V}\rvert$	0	9	3	0	6	6	0	15	9

For all parameters, therefore, values from series (A) generally lie between those from (B) and (C), the tendency being least marked for $|\overline{V}|$, q, $\sigma_{(\theta)}/|\overline{V}|$, for which, however, the deviations are such as to strengthen the evidence in favour of the representativeness of series (A).

Monthly and annual mean wind speeds ($\overline{V}_{(S)}$) for the several series are given overleaf.

Table II(c)(2). Mean Monthly and Annual Windspeeds at Ocean Weather Ship "J", 1950-52 and 1953-55

(knots)

Series	Jan.	Feb.	March	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
						1950-52							
"B"	24.4	21.0	22.0	17.9	15.6	15.0	15.7	18.5	18.3	23.1	20.7	24.5	19.7
"C"	23.4	21.0	22.1	18.1	15.9	15.7	15.5	18.2	19.0	23.2	21.4	23.9	19.8
"A"	24.0	20.5	22.0	17.9	15.9	15.2	15.6	18.6	18.7	22.9	21.1	23.9	19.7
						1953-55							
"B"	22.2	23.0	20.9	19.0	18.1	16.5	15.4	17.2	21.1	19.3	22.8	25.7	20.1
"C"	20.4	25.2	19.0	18.5	17.2	15.7	14.7	16.7	22.1	18.4	20.9	23.6	19.4
"A"	20.8	23.7	19.6	18.4	17.2	16.0	15.2	16.9	21.6	18.9	21.8	24.3	19.5

Regarding any systematic difference between wind speeds at 0300 and 1500 GMT, it will be noted that

$$\overline{V}_{(S)(03)} > \overline{V}_{(S)(15)}$$

on four occasions in 1950-52 and ten occasions in 1953-55, or fourteen in all. There is therefore a balance of evidence for regarding series (B) and (C) as rather less representative than (A), and certainly nothing is lost by basing analyses upon the more detailed series.

Computed values of all the important parameters are set out in Appendix II (d) Section 1(i), from which it is possible to judge the extent to which observations at either of the hours 0300 GMT or 1500 GMT may serve to give an estimate of the monthly mean value of the selected parameter.

Section 2. The "Northice" data.

Mean wind speeds by month of a given name, for observations at particular hours, or combinations of hours, are set out below, from which it will be seen that the effect of grouping upon the result is very small.

Table II(c)(3). Comparison of Mean Wind Speeds for Periods indicated:

(i) derived from grouped frequency table

(ii) as given by Hamilton & Rollitt (1957)

Series	(knots)		Series	(knots)	
Jan.(1953) and 1954)	(i)	(ii)	July (1953)	(i)	(ii)
0001 plus 0600 GMT	16.3	16.4	0600 GMT	12.7	12.7
1200 plus 1800 GMT	14.9	15.0	1200 plus 1800 GMT	14.2	14.3
Feb.(1953 and 1954)			August (1953)		
0001 plus 0600 GMT	15.7	15.9	0600 GMT	15.5	15.6
1200 plus 1800 GMT	15.8	16.0	1200 plus 1800 GMT	15.1	15.1
March (1953 and 1954)			September (1953)		
0001 plus 0600 GMT	15.4	15.5	0600 GMT	13.5	13.5
1200 plus 1800 GMT	15.6	15.8	1200 plus 1800 GMT	14.1	14.0
April (1953 and 1954)			October (1953)		
0001 plus 0600 GMT	15.6	15.5	0600 GMT	15.0	15.0
1200 plus 1800 GMT	14.7	14.6	1200 plus 1800 GMT	14.9	14.7
May (1953 and 1954)			November (1952 and 1953)		
0600 GMT	12.5	12.5	0600 GMT	16.9	16.7
1200 plus 1800 GMT	11.6	11.5	1200 plus 1800 GMT	16.4	16.4
June			December (1952 and 1953)		
0001 GMT (1954) plus 0600 GMT (1953 and 1954)	9.6	9.6	0001 plus 0600 GMT	14.7	14.7
(1953 and 1954) 1200 plus 1800 GMT	10.1	10.2	1200 plus 1800 GMT	15.6	15.4

The data selected for detailed graphical analysis consists of "seasonal" values of observation for 0001 (or in their absence those for 2100 GMT), 0600, 1200 and 1800 GMT combined. Separate tables are however available for sub-series in which data for the first two hours, and for the second two hours, are amalgamated.

Hamilton (1958), in a detailed study analysing the mid-summer conditions (1 June to 15 July 1953), found no evidence of any diurnal variation in either the speed or the direction of the surface wind at this station.

It would appear from the data in Appendix II(d) Sec.2, that the seasonal variation is mainly one of changes in the magnitude of the wind speed. Scrutiny of the frequencies by direction indicate little variation between the two sub-series.

From the evidence outlined above there would seem no reason to doubt but that a fully representative sample of wind distribution for the 24 hours is given by pooling observations for the four evenly spaced hours.

Section 3. The "Bell Rock" data

In the Table on page 6 (Table II(c)(4)), estimates of the mean monthly wind speeds as obtained by various procedures are set out. Values under (iv) are to be regarded as the true values. The "(a)" series has been derived from data grouped into speed categories as already mentioned (p.128) and the "(b)" series directly from the ungrouped data. In the spring and summer months an appreciable difference between the estimates based upon the 0400 GMT and the 1600 GMT observations may be noted, associated (see Appendix II(d) Section 3) with a directional shift, but in "autumn" and "winter" little diurnal variation in wind velocity is apparent.

The relatively small differences between results under "(iii)" and "(iv)" indicate that, if long period monthly mean speeds are required, the mean of the "0400 GMT" and "1600 GMT" readings give a tolerable approximation; furthermore, in "autumn" and "winter" a fairly close estimate of the period monthly mean wind velocity is provided by the average of once daily

Table II(c)(4). Monthly Mean and Speeds, Bell Rock, December 1950 to November 1955, based upon:

(i)(a) hourly mean speeds for hour ending 0400 GMT (grouped data)
(i)(b) hourly mean speeds for hour ending 0400 GMT (ungrouped data)
(ii)(a) hourly mean speeds for hour ending 1600 GMT (grouped data)
(ii)(b) hourly mean speeds for hour ending 1600 GMT (ungrouped data)
(iii)(a) combined data from series (i)(a) and (ii)(a)
(iii)(b) combined data from series (i)(b) and (ii)(b)
(iv) mean, daily displacement or "run-of-wind" (ungrouped data)

	(i) "0400 GMT"		(ii) "1600 GMT"		(iii) Combined Series		(iv) "all hours"
	(i)(a)	(i)(b)	(ii)(a)	(ii)(b)	(iii)(a)	(iii)(b)	
Jan.	18.5	18.6	18.9	18.9	18.7	18.7	18.8
Feb.	17.4	17.3	17.0	17.2	17.2	17.3	17.2
March	14.9	14.7	14.7	14.7	14.8	14.7	14.6
April	13.8	13.6	15.5	15.4	14.6	14.5	14.0
May	13.8	13.9	13.7	14.0	13.8	13.9	13.9
June	12.6	12.4	13.9	13.9	13.2	13.2	12.9
July	10.5	10.5	12.1	12.2	11.3	11.3	10.9
Aug.	12.9	12.8	14.4	14.6	13.7	13.7	13.4
Sept.	15.2	15.3	16.2	16.0	15.7	15.7	15.7
Oct.	18.0	18.0	17.1	17.1	17.5	17.5	17.4
Nov.	18.5	18.2	18.6	18.5	18.6	18.4	18.6
Dec.	19.9	19.9	19.4	19.5	19.7	19.7	19.5

readings at either of the two observation hours.

A comparison of the columns headed "(a)" with those headed "(b)" shows that the effect of the purely arithmetic device of grouping on mean wind speed is quite small.

The study by Goldie (1935) of the winds at Bell Rock during the period September 1929 to August 1933, based upon observations for the 24 hours, presents results of immediate interest.

Below are given, on a seasonal basis, the ratios of the mean speeds at 0400 and 1600 GMT to that based on 24-hour data according to Goldie (1935), and, in brackets, the corresponding values for the rather longer series of the present investigation.

"Spring" :	$\overline{V}_{(04)}/\overline{V}_{(24)}$ = 0.971 (0.997)	;	$\overline{V}_{(16)}/\overline{V}_{(24)}$ = 1.058 (1.030)
"Summer" :	$\overline{V}_{(04)}/\overline{V}_{(24)}$ = 1.000 (0.966)	;	$\overline{V}_{(16)}/\overline{V}_{(24)}$ = 1.056 (1.086)
"Autumn" :	$\overline{V}_{(04)}/\overline{V}_{(24)}$ = 0.996 (0.999)	;	$\overline{V}_{(16)}/\overline{V}_{(24)}$ = 0.991 (1.003)
"Winter" :	$\overline{V}_{(04)}/\overline{V}_{(24)}$ = 1.013 (1.008)	;	$\overline{V}_{(16)}/\overline{V}_{(24)}$ = 0.947 (0.998)

Goldie also presents, for the "summer", vector diagrams showing the position, hour by hour throughout the 24 hours, of the mean vector wind. He finds that the mean wind at 1600 GMT is from a direction slightly west of south, and is backed by about 70° from the mean direction at 0400 GMT. It is also evident from his Fig. 28, that the vector mean of the mean velocities at 0400 GMT and 1600 GMT differs little from the resultant for the 24 hours. This diurnal change in wind velocity in "summer" at Bell Rock is attributed by Goldie to a persistent tendency for inflow towards the central mountain region in the afternoons (associated with convection over the mountains), and an outflow during the night.

On the basis of the evidence presented, it is claimed that the sample of observations used may be regarded as representative.

Section 4. The Lerwick data.

It will be noted from Table II(c)(5) that the true value of the monthly mean speed shown in col. (iv) is closely given by the mean of the speeds at 0400 and 1600 GMT, in col. (iii): a portion but, judging from the analysis of the Bell Rock records, only a small portion of such differences as appear, should be attributed to grouping.

As is to be expected, the diurnal variation, as expressed by the differences between col. (i) and col. (ii) is greater in the summer half-year than in the winter half, although, December apart, it is only approximately true that a satisfactory sample of the mean wind for any month of given name is provided by either the 0400 or the 1600 GMT - in contrast to Bell Rock.

On a seasonal basis (see Section 4 of Appendix II(d)) the diurnal variation in wind speed persists, but there is only a small associated change in direction even in "spring" and "summer" - again in contrast to Bell Rock.

Table II(c)(5). Monthly Mean Surface Wind-Speeds, Lerwick, 1951-55, based upon:

(i) hourly mean speeds for hour ending 0400 GMT (grouped data)
(ii) hourly mean speeds for hour ending 1600 GMT (grouped data)
(iii) combined data of series (i) and (ii) (grouped data)
(iv) mean daily displacement or "run-of-wind" (ungrouped data)

Month	Series (i)	(ii)	(iii)	(iv)
	meters per second			
Jan.	7.65	8.20	7.93	7.97
Feb.	8.43	8.75	8.59	8.68
March	7.90	8.83	8.37	8.16
April	6.27	8.03	7.15	7.04
May	5.23	6.54	5.90	5.88
June	5.06	6.68	5.87	5.83
July	5.20	6.31	5.78	5.74
Aug.	5.11	6.22	5.67	5.68
Sept.	6.53	7.57	7.05	6.84
Oct.	7.50	8.08	7.79	8.01
Nov.	8.12	8.79	8.46	8.40
Dec.	8.73	8.87	8.80	8.89

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Appendix II (d)

Basic Parameters for the four Series of Surface Wind Observations

Some of the main parameters are listed below.

Most of the symbols are defined in the Glossary (p. iv), and others are defined as they arise.

Section 1. Ocean weather ship "Juliet" (mean position $52\frac{1}{2}^{\circ}$N, 20°W) 1950-55 (inclusive)

Values, month by month, are given separately for the two three-year periods, viz. 1950-52 and 1953-55 for:-

"A" - "all hours" (viz. the standard eight synoptic hours)

"B" - "0300" GMT only

"C" - "1500" GMT only

Data for series "A" are then aggregated to produce values for the conventional "seasons", viz.

"Spring" - March-April-May;

"Summer" - June-July-August;

"Autumn" - September-October-November

"Winter" - December-January-February.

for each of the two three-year periods.

Finally, seasonal results are set out for the complete six-year period.

q is expressed in percent, ϕ in degrees from north, the unit of speed is the knot.

(i) Results, month-by-month

Series	Parameters									
	$\bar{V}_{(s)}$	$\lvert\bar{V}\rvert$	ϕ	s	σ_o	σ_N	σ_E	q	$\sigma_o/\lvert\bar{V}\rvert$	$s/\bar{V}_{(s)}$
					January, 1950-52					
"A"	24.0	14.8	255	10.2	21.5	16.4	13.9	61.7	1.45	0.426
"B"	24.4	15.0	255	9.9	21.6	17.2	13.2	61.6	1.44	0.405
"C"	23.4	14.2	254	10.3	21.2	15.6	14.5	60.5	1.50	0.438
					January, 1953-55					
"A"	20.8	5.5	213	10.0	22.4	14.7	16.8	26.5	4.07	0.480
"B"	22.2	6.9	217	12.0	24.2	14.6	19.4	31.2	3.50	0.539
"C"	20.4	6.4	210	9.7	21.6	14.2	16.3	31.7	3.35	0.476
					February, 1950-52					
"A"	20.5	8.7	259	9.5	20.9	14.6	14.9	42.2	2.41	0.465
"B"	21.0	9.4	253	10.7	21.6	16.0	14.5	44.7	2.30	0.509
"C"	21.0	9.6	258	8.77	20.7	14.0	15.2	45.6	2.16	0.417
					February, 1953-55					
"A"	23.7	8.5	259	11.8	25.0	17.1	18.3	35.9	2.94	0.497
"B"	23.0	9.1	257	12.1	24.4	16.0	18.4	39.6	2.68	0.528
"C"	25.2	9.4	257	11.4	26.0	18.2	18.5	37.3	2.77	0.451
					March, 1950-52					
"A"	22.0	2.7	219	9.2	23.7	16.7	16.9	12.4	8.71	0.419
"B"	22.0	1.6	218	9.2	23.8	17.4	16.2	7.2	15.0	0.420
"C"	22.1	3.7	212	9.7	23.8	16.1	17.6	16.6	6.52	0.440

Series	Parameters									
	$\overline{V}(s)$	$\lvert\overline{\mathbf{V}}\rvert$	ϕ	s	σ_0	σ_N	σ_E	q	$\sigma_0/\lvert\overline{\mathbf{V}}\rvert$	$s/\overline{V}(s)$
					March, 1953-55					
"A"	19.6	4.0	189	8.7	21.0	15.5	14.2	20.3	5.31	0.443
"B"	20.9	5.2	185	8.7	22.0	16.4	14.6	24.7	4.27	0.415
"C"	19.0	4.5	198	9.1	20.5	14.9	14.2	23.5	4.62	0.479
					April, 1950-52					
"A"	17.9	8.5	265	9.2	18.2	13.0	12.8	47.6	2.14	0.510
"B"	17.9	9.4	268	8.4	17.5	12.2	12.5	52.3	1.86	0.470
"C"	18.1	8.8	260	9.7	18.6	13.3	13.0	48.5	2.11	0.533
					April, 1953-55					
"A"	18.4	5.1	224	8.7	19.7	14.2	13.7	27.5	3.89	0.473
"B"	19.0	5.5	229	8.8	20.2	14.6	14.0	29.2	3.64	0.463
"C"	18.5	5.6	225	7.8	19.3	14.0	13.7	30.1	3.47	0.422
					May, 1950-52					
"A"	15.9	2.5	64	7.7	17.5	13.8	10.8	15.5	7.11	0.484
"B"	15.6	1.9	56	7.3	17.1	13.4	10.6	12.5	8.80	0.470
"C"	15.9	2.6	84	7.6	17.4	13.9	10.5	16.1	6.83	0.479
					May, 1953-55					
"A"	17.2	3.9	205	8.7	18.9	13.3	13.5	22.3	4.92	0.506
"B"	18.1	4.5	199	9.0	19.7	13.4	14.5	24.7	4.41	0.498
"C"	17.2	4.0	207	8.5	18.7	12.9	13.6	23.4	4.66	0.492
					June, 1950-52					
"A"	15.2	6.2	243	7.2	15.7	11.4	10.9	40.5	2.54	0.475
"B"	15.0	6.7	244	6.9	15.1	10.7	10.6	45.0	2.23	0.460
"C"	15.7	7.0	238	7.4	15.9	11.9	10.5	44.7	2.26	0.474

Series	Parameters									
	$\overline{V}(s)$	$\lvert\overline{\mathbf{V}}\rvert$	ϕ	s	σ_0	σ_N	σ_E	q	$\sigma_0/\lvert\overline{\mathbf{V}}\rvert$	$s/\overline{V}(s)$
					June, 1953-55					
"A"	16.0	6.5	251	7.2	16.3	12.5	10.6	40.6	2.51	0.450
"B"	16.5	6.6	246	7.5	16.9	13.1	10.7	39.8	2.57	0.452
"C"	15.7	7.0	252	6.7	15.6	11.4	10.6	44.5	2.23	0.430
					July, 1950-52					
"A"	15.6	9.7	256	7.1	14.2	10.8	9.1	62.1	1.46	0.458
"B"	15.7	9.5	254	7.1	14.4	11.2	9.0	60.6	1.51	0.448
"C"	15.5	9.9	258	7.1	13.9	10.3	9.3	64.0	1.40	0.460
					July, 1953-55					
"A"	15.2	9.5	266	7.2	13.8	10.1	9.4	62.6	1.46	0.471
"B"	15.4	9.5	264	7.9	14.4	10.6	9.8	61.9	1.51	0.511
"C"	14.7	9.4	266	6.9	13.2	9.6	9.2	64.1	1.40	0.470
					August, 1950-52					
"A"	18.6	12.0	288	9.5	17.0	12.5	11.6	64.9	1.41	0.510
"B"	18.5	12.5	288	8.6	16.1	11.7	11.1	67.7	1.29	0.467
"C"	18.2	11.7	284	9.6	16.9	13.0	10.7	64.7	1.44	0.529
					August, 1953-55					
"A"	16.9	8.5	241	8.5	16.9	12.2	11.8	50.0	2.00	0.502
"B"	17.2	8.7	241	8.7	17.1	13.0	11.1	50.9	1.96	0.504
"C"	16.7	8.2	235	7.9	16.6	12.2	11.2	49.1	2.02	0.474
					September, 1950-52					
"A"	18.7	10.2	266	9.3	18.3	12.9	12.9	54.3	1.80	0.495
"B"	18.3	10.3	260	9.3	17.8	11.9	13.2	56.1	1.73	0.508
"C"	19.0	10.1	262	8.7	18.3	13.3	12.5	53.1	1.81	0.456

Series	Parameters									
	$\overline{V}(s)$	$\lvert\overline{V}\rvert$	ϕ	s	σ_0	σ_N	σ_E	q	$\sigma_0/\lvert\overline{V}\rvert$	$s/\overline{V}(s)$
					September, 1953-55					
"A"	21.6	14.5	262	8.9	18.4	13.6	12.4	66.9	1.27	0.413
"B"	21.1	14.6	260	8.9	17.6	13.0	11.9	69.3	1.20	0.420
"C"	22.1	14.1	259	8.6	19.1	14.0	13.0	63.5	1.36	0.386
					October, 1950-52					
"A"	22.9	11.4	239	11.5	22.9	15.3	17.1	49.7	2.01	0.501
"B"	23.1	11.0	241	10.5	22.9	14.8	17.5	47.5	2.08	0.456
"C"	23.2	12.6	240	12.2	23.0	15.6	16.9	54.1	1.83	0.524
					October, 1953-55					
"A"	18.9	7.7	254	8.1	19.6	14.3	12.6	40.8	2.48	0.432
"B"	19.3	8.9	255	8.3	19.0	14.4	12.4	46.0	2.15	0.429
"C"	18.4	6.6	244	8.1	19.0	14.1	12.7	36.0	2.87	0.441
					November, 1950-52					
"A"	21.1	7.5	298	9.6	21.9	14.3	16.6	35.5	2.93	0.455
"B"	20.7	7.8	293	10.0	21.7	13.7	16.8	37.5	2.79	0.482
"C"	21.4	7.2	293	9.1	22.1	15.0	16.2	33.7	3.06	0.423
					November, 1953-55					
"A"	21.8	9.1	245	9.6	22.0	15.9	15.2	41.6	2.43	0.441
"B"	22.8	8.5	249	10.6	23.7	17.4	16.0	37.2	2.79	0.464
"C"	20.9	10.5	238	9.3	20.3	15.2	13.5	50.4	1.93	0.445

Series	Parameters									
	$\overline{V}(s)$	$\lvert\overline{\mathbf{V}}\rvert$	ϕ	s	σ_0	σ_N	σ_E	q	$\sigma_0/\lvert\overline{\mathbf{V}}\rvert$	$s/\overline{V}(s)$
					December, 1950-52					
"A"	23.9	13.0	279	8.8	21.9	15.9	15.0	54.5	1.68	0.369
"B"	24.5	13.3	280	9.2	22.5	16.5	15.4	54.4	1.69	0.375
"C"	23.9	13.5	281	8.1	21.3	15.6	14.5	56.7	1.57	0.339
					December, 1953-55					
"A"	24.3	13.6	236	11.5	23.2	16.1	16.6	56.1	1.70	0.473
"B"	25.7	14.7	238	12.2	24.4	16.3	18.1	57.3	1.65	0.474
"C"	23.6	13.9	237	11.7	22.3	15.9	15.7	59.1	1.60	0.496

(ii) Results, by season, for each three-year period. ("all hours" only)

$\overline{V}(s)$	$\lvert\overline{\mathbf{V}}\rvert$	ϕ	s	σ_0	σ_N	σ_E	q	$\sigma_0/\lvert\overline{\mathbf{V}}\rvert$	$s/\overline{V}(s)$
				"Spring"					
				1950-52					
18.6	2.7	258	9.1	20.5	14.6	14.4	14.4	7.66	0.489
				1953-55					
18.4	4.1	206	8.7	20.0	14.4	13.8	22.5	4.82	0.475
				"Summer"					
				1950-52					
16.5	8.8	267	8.1	16.1	11.9	10.8	53.5	1.83	0.496
				1953-55					
16.0	8.04	253	7.7	15.9	11.7	10.7	50.0	1.98	0.479
				"Autumn"					
				1950-52					
21.0	8.9	263	10.3	21.6	14.8	15.7	42.7	2.41	0.493
				1953-55					
20.7	10.3	255	9.0	20.0	14.6	13.7	50.0	1.93	0.433

$\overline{V(s)}$	$\lvert\overline{V}\rvert$	ϕ	s	σ_O	σ_N	σ_E	q	$\sigma_O/\lvert\overline{V}\rvert$	$s/\overline{V(s)}$
				"Winter"					
				1950-52					
22.8	12.0	264	9.7	21.7	15.8	14.8	52.6	1.81	0.425
				1953-55					
22.9	8.9	238	11.2	23.9	16.2	17.6	38.8	2.69	0.489

(i) Results, by seasons, for the complete six-year period. ("all hours" only)

$\overline{V(s)}$	$\lvert\overline{V}\rvert$	ϕ	s	σ_O	σ_N	σ_E	q	$\sigma_O/\lvert\overline{V}\rvert$	$s/\overline{V(s)}$
				"Spring"					
18.5	3.1	227	8.9	20.3	14.6	14.1	16.7	6.55	0.482
				"Summer"					
16.2	8.4	260	7.9	16.0	11.9	10.8	51.4	1.94	0.488
				"Autumn"					
20.8	9.6	259	9.7	20.9	14.7	14.8	46.2	2.17	0.470
				"Winter"					
22.9	10.2	253	10.5	23.0	16.1	16.4	44.5	2.26	0.458

Section 2

"Northice" series (78°04'N, 38°29'W., height 2,343 m. above msl.)

Period: 1 November 1952 to 15 July 1954; and observations for 0001GMT (or in their absence those for 2100GMT), 0600GMT, combined with those for 1200 GMT and 1800 GMT.

All units as for the O.W.S."J" data.

$\overline{V}(s)$	$\lvert\overline{V}\rvert$	ϕ	s	σ_o	σ_N	σ_E	q	$\sigma_o/\lvert\overline{V}\rvert$	$s/\overline{V}(s)$
				Spring					
14.2	13.3	269	6.00	7.75	4.63	6.20	93.8	0.583	0.424
				Summer					
12.2	9.30	272	5.70	9.70	5.67	7.87	76.4	1.043	0.468
				Autumn					
15.6	14.0	271	5.71	8.90	6.03	6.54	89.9	0.635	0.366
				Winter					
15.5	13.3	262	5.74	9.89	5.75	8.04	85.5	0.745	0.370

Section 3

Bell Rock series (56°26'N, 2°24'W., 130 ft. above m.s.l.)

Period: 1 December 1950 to 30 November 1955 (owing to damage to the anemometer in December 1955, the series was completed by using the observations for December 1950).

"B" - mean hourly value for the hour ending 0400GMT.

"C" - mean hourly value for the hour ending 1600GMT.

"M" - mean hourly value for series "B" and "C" combined.

All units as for the O.W.S."J" data.

	$\overline{V}(s)$	$\lvert\overline{V}\rvert$	ϕ	s	σ_o	σ_N	σ_E	q	$\sigma_o/\lvert\overline{V}\rvert$	$s/\overline{V}(s)$
					"Spring"					
"M"	14.4	1.51	261	8.52	16.7	10.9	12.6	10.5	11.04	0.592
"B"	14.2	2.24	307	8.85	16.5	10.7	12.6	15.8	7.38	0.625
"C"	14.6	2.28	212	8.17	16.6	10.8	12.6	15.6	7.27	0.559

	$\overline{V}(s)$	$\lvert\overline{V}\rvert$	ϕ	s	σ_0	σ_N	σ_E	q	$\sigma_0/\lvert\overline{V}\rvert$	$s/\overline{V}(s)$
					"Summer"					
"M"	13.0	2.92	256	7.45	14.7	9.28	11.4	22.5	5.02	0.574
"B"	12.2	3.63	273	7.65	14.0	8.79	10.8	29.7	3.84	0.626
"C"	13.7	3.65	219	7.17	15.1	9.29	11.9	26.6	4.13	0.523
					"Autumn"					
"M"	17.5	7.49	258	8.72	18.1	12.7	12.9	42.7	2.42	0.497
"B"	17.5	7.74	259	8.34	17.8	12.9	12.3	44.2	2.30	0.477
"C"	17.6	7.21	256	9.03	18.4	12.6	13.4	41.0	2.55	0.514
					"Winter"					
"M"	18.6	7.99	277	9.61	19.3	13.5	13.9	43.0	2.42	0.517
"B"	18.7	8.71	277	9.85	19.2	13.5	13.7	46.7	2.21	0.528
"C"	18.5	7.33	278	9.41	19.4	13.4	14.0	39.2	2.65	0.509

Section 4. Lerwick Observatory series (60°8'N, 1°11'W. 269 ft.above msl).

Period: 1 January 1951 - 31 December 1955.

"B" - mean hourly values for the hour ending 0400GMT.

"C" - mean hourly values for the hour ending 1600GMT.

"M" - mean hourly values for series "B" and "C" combined.

The unit of speed is the meter per sec.

	$\overline{V}(s)$	$\lvert\overline{V}\rvert$	ϕ	s	σ_0	σ_N	σ_E	q	$\sigma_0/\lvert\overline{V}\rvert$	$s/\overline{V}(s)$
					"Spring"					
"M"	7.14	1.08	267	4.23	8.23	5.98	5.65	15.1	7.63	0.593
"B"	6.46	1.03	269	4.28	7.68	5.54	5.32	16.0	7.43	0.663
"C"	7.82	1.13	266	4.07	8.74	6.39	5.96	14.4	7.74	0.521

	$\overline{V}(s)$	$\lvert\overline{\mathbf{V}}\rvert$	ϕ	s	σ_0	σ_N	σ_E	q	$\sigma_0/\lvert\overline{\mathbf{V}}\rvert$	$s/\overline{V}(s)$
					"Summer"					
"M"	5.75	1.62	281	3.66	6.62	4.76	4.60	28.2	4.09	0.637
"B"	5.10	1.53	285	3.62	6.06	4.36	4.20	30.0	3.96	0.709
"C"	6.40	1.71	278	3.59	7.13	5.11	4.98	26.7	4.17	0.561
					"Autumn"					
"M"	7.78	2.32	236	4.56	8.71	6.37	5.94	29.8	3.76	0.585
"B"	7.40	2.06	234	4.62	8.48	6.03	5.96	27.8	4.11	0.625
"C"	8.16	2.48	239	4.46	8.95	6.69	5.92	30.4	3.61	0.547
					"Winter"					
"M"	8.45	2.77	248	5.23	9.55	7.46	5.96	32.7	3.43	0.619
"B"	8.25	2.71	247	5.21	9.38	7.22	5.98	32.8	3.46	0.632
"C"	8.65	2.76	248	5.23	9.73	7.68	5.97	31.9	3.52	0.605